

# केंद्रीय विद्यालय संगठन क्षेत्रीय कार्यालय रायपुर

**Kendriya Vidyalaya Sangathan Regional Office Raipur**



# Mathematics



# Class – XII

# Question Bank Term- II

## Session:2021-22

# केंद्रीय विद्यालय संगठन क्षेत्रीय कार्यालय रायपुर

## Kendriya Vidyalaya Sangathan Regional Office Raipur

### MESSAGE FROM DUPUTY COMMISSIONER



It is a matter of great pleasure for me to publish study material for different subjects of classes X and XII for Raipur Region. Getting acquainted and familiarized with the recent changes in curriculum and assessment process made by CBSE vide Circular No. 51 and 53 issued in the month of July 2021 will help students to prepare themselves better for the examination. Sound and deeper knowledge of the Units and Chapters is must for grasping the concepts, understanding the questions. Study materials help in making suitable and effective notes for quick revision just before the examination.

Due to the unprecedented circumstances of COVID-19 pandemic the students and the teachers are getting very limited opportunity to interact face to face in the classes. In such a situation the supervised and especially prepared value points will help the students to develop their understanding and analytical skills together. The students will be benefitted immensely after going through the question bank and practice papers. The study materials will build a special bond and act as connecting link between the teachers and the students as both can undertake a guided and experiential learning simultaneously. It will help the students develop the habit of exploring and analyzing the **Creative & Critical Thinking Skills**. The new concepts introduced in the question pattern related to case study, reasoning and ascertain will empower the students to take independent decision on different situational problems. The different study materials are designed in such a manner to help the students in their self-learning pace. It emphasizes the great pedagogical dictum that '*everything can be learnt but nothing can be taught*'. The self-motivated learning as well as supervised classes will together help them achieve the new academic heights.

I would like to extend my sincere gratitude to all the principals and the teachers who have relentlessly striven for completion of the project of preparing study materials for all the subjects. Their enormous contribution in making this project successful is praiseworthy.

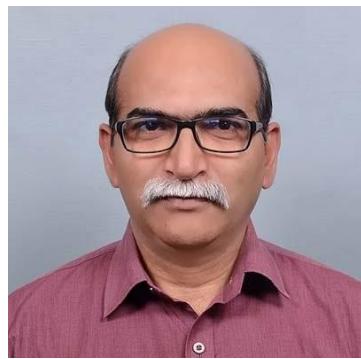
Happy learning and best of luck!

Vinod Kumar  
(Deputy Commissioner)

# केंद्रीय विद्यालय संगठन क्षेत्रीय कार्यालय रायपुर

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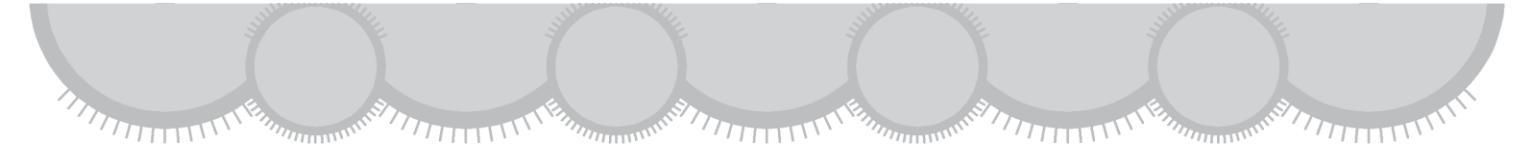
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Deputy Commissioner  
KVS RO Raipur



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**CLASS-XII**  
**MATHEMATICS (Code No. 041) Session – 2021-22**  
**TERM – II**

No.	Units	Marks
III	Calculus	18
IV	Vectors and Three-Dimensional Geometry	14
VI	Probability	8
	<b>Total</b>	40
	<b>Internal Assessment</b>	10
	<b>Total</b>	50

***Unit-III: Calculus***

**1. Integrals**

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}$$

$$\int \frac{px + q}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^2+bx+c}} dx, \int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2 - a^2} dx$$

**2. Applications of the Integrals**

Applications in finding the area under simple curves, especially lines, parabolas; area of circles /ellipses (in standard form only) (the region should be clearly identifiable).

**3. Differential Equations**

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree of the type:

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right).$$

Solutions of linear differential equation of the type:

$$\frac{dy}{dx} + Py = Q \text{ where } p \text{ and } q \text{ are functions of } x \text{ or constant.}$$

## ***Unit-IV: Vectors and Three-Dimensional Geometry***

### **1. Vectors**

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

### **2. Three - dimensional Geometry**

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Distance of a point from a plane.

## ***Unit-VI: Probability***

### **1. Probability**

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution.

<b>Internal Assessment</b>	<b>10 Marks</b>
Periodic Test	5 Marks
Mathematics Activities: Activity file record +Term end assessment of one activity & Viva	5 Marks

## **CHAPTER: 7 INTEGRALS**

### **SHORT ANSWER TYPE QUESTIONS-I (SA-I) (2 MARKS)**

Evaluate:

Q.1  $\int (x^{3/2} + 2e^x - \frac{1}{x}) dx$

Q.2  $\int \frac{(x^3 - x^2 + x - 1)}{x-1} dx$

Q.3  $\int \frac{(2-3\sin x)}{\cos^2 x} dx$

Q.4  $\int \frac{(\tan 4\sqrt{x} \sec 2\sqrt{x})/\sqrt{x}}{\sqrt{x}} dx$

Q.5  $\int \frac{1}{x+x\log x} dx$

Q.6  $\int \sin(ax+b) \cos(ax+b) dx$

Q.7  $\int \frac{x^2}{(2+3x^2)^2} dx$

Q.8  $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}} dx$

Q.9 Find the antiderivative F of f defined by  $f(x) = 4x^3 - 6$ , where  $F(0) = 3$ .

### **SHORT ANSWER TYPE QUESTIONS-II (SA-II) (3 MARKS)**

Q.1  $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

Q.2  $\int \frac{1}{1+\cot x} dx$

Q.3  $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$

Q.4  $\int \frac{3x}{1+2x^4} dx$

Q.5  $\int \log x dx$

Q.6  $\int \sqrt{(x^2 + 2x + 5)} dx$

Q.7  $\int_0^{\pi} (\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2})$

Q.8  $\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

Q.9  $\int_0^1 x(1-x)^n dx$

Q.10  $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$

Q.11  $\int_4^9 \frac{\sqrt{x}}{(30-x^{\frac{3}{2}})^2} dx$

### LONG ANSWER TYPE QUESTIONS (4 MARKS)

Solve the following:

Q.1  $\int \sin^3 x \cos^2 x \, dx.$

Q.2  $\int \frac{\sin x}{\sin(x+\alpha)} \, dx.$

Q.3  $\int \frac{\sqrt{\tan x}}{\sin x \cos x} \, dx$

Q.4  $\int \cos 2x \cos 4x \cos 6x \, dx$

Q.5  $\int \cos^4 2x \, dx$

Q.6  $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} \, dx$

Q.7  $\int \frac{1}{\cos(x-a)\cos(x-b)} \, dx$

Q.8  $\int \frac{x^2+1}{x^2-5x+6} \, dx$

Q.9  $\int \frac{x^2}{(x^2+1)(x^2+4)} \, dx$

Q.10  $\int \frac{(3\sin \phi - 2)\cos \phi}{5 - \cos^2 \phi - 4\sin \phi} \, d\phi$

Q.11  $\int \frac{1}{x(x^n+1)} \, dx$

Q.12  $\int x \cdot \sin^{-1} x \, dx$

Q.13  $\int (\sin^{-1} x)^2 \, dx$

Q.14  $\int \frac{(x-3)^2 e^x}{(x-1)^3} \, dx$

Q.15  $\int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} \cdot \cos^5 \phi \, d\phi$

Q.16  $\int_1^2 \left( \frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} \, dx$

Q.17  $\int_{-1}^2 |x^3 - x| \, dx$

Q.18  $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$

Q.19  $\int [\log(\log x) + \frac{1}{(\log x)^2}] \, dx$

Q.20  $\int (\sqrt{\tan x} + \sqrt{\cot x}) \, dx$

Q.21  $\int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

Q.22  $\int \sqrt{\tan x} \, dx$

Q.23  $\int_{-1}^2 (|x+1| + |x| + |x-1|) \, dx$

$$Q.24 \quad \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4 \sin^2 x} dx$$

$$Q.25 \quad \int_0^1 x (\tan^{-1} x)^2 dx$$

$$Q.26 \quad \tan^8 x \sec^4 x dx$$

$$Q.27. \quad \int \frac{x^2}{(x-1)(x+1)^2} dx$$

$$Q.28 \quad \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

$$Q.29 \quad \int \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx$$

$$Q.30 \quad \int e^x \sin x dx$$

$$Q.31 \quad \int_{-1}^{3/2} |x \sin x| dx$$

$$Q.32 \quad \int \frac{\sec x}{1 + \operatorname{cosec} x} dx$$

$$Q.33 \quad \int_{-1}^1 \frac{x + |x| + 1}{x^2 + 2|x| + 1} dx$$

$$Q.34 \quad \text{Show that } \int_{a+c}^{b+c} f(x) dx = \int_a^b f(x+c) dx$$

Q.35 If  $f$  and  $g$  are continuous functions in  $[0,1]$  satisfying  $f(x) = f(a-x)$

and  $g(x) + g(a-x) = a$  then show that  $\int_0^a f(x)g(x) dx = \frac{a}{2} \int_0^a f(x) dx$

$$Q.36 \quad \text{Show that } \int_0^{\frac{\pi}{2}} f(\sin 2x) \sin x dx = \sqrt{2} \int_0^{\frac{\pi}{4}} f(\cos 2x) \cos x dx$$

$$Q.37 \quad \text{Evaluate } \int \frac{\cos 5x + \cos 4x}{1 - 2 \cos 3x} dx$$

## CASE STUDY BASED QUESTIONS

Q.1 Modulus function  $|x|$  is defined as follows  $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$ . Also, if for any function

$f(x)$ , we have  $\int_a^b f(x)dx = \int_a^{c_1} f(x)dx + \int_{c_1}^{c_2} f(x)dx + \dots \dots + \int_{c_n}^b f(x)dx$ .

where  $a < c_1 < c_2 < c_3 \dots \dots < c_n < b$

Based on above information answer the following questions:

(i) Find value of  $\int_0^1 |3x - 2|dx$ .

(ii) Find value of  $\int_0^{\pi} |\cos x| dx$ .

Q.2 A function can be divided into odd and even functions on the basis of following

$f(x)$  is odd if  $f(-x) = -f(x)$

$f(x)$  is even if  $f(-x) = f(x)$

Ramesh wants to solve integration questions using above concept and properties of integration. Help Ramesh in finding the solutions to following questions

(i) What is the value of  $\int_{-2}^2 [f(x) - f(-x)]dx$  if  $f(x)$  is an even function

(ii) What is the value of  $\int_{-2}^2 [f(x) + f(-x)] dx$  if  $f(x)$  is an odd function

(iii) Evaluate  $\int_{-\pi}^{\pi} \frac{x}{1+\cos^2 x} dx$

Q.3 If  $f(x)$  is a continuous function defined on  $[0, a]$  then  $\int_0^a f(x)dx = \int_0^a f(a - x)dx$

On basis of above information answer the following questions

(i)  $\int_0^a \frac{f(x)}{f(x)+f(a-x)} dx$  will be equal to ..... .

(ii)  $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx = \dots \dots \dots$

## ANSWERS KEY SA-I (2 MARKS)

1.  $\frac{2}{5}x^{\frac{5}{2}} + 2e^x - \log|x| + C$

2.  $\frac{x^3}{3} + x + C$

3.  $2 \tan x - 3 \sec x + C$

4.  $\frac{2}{5} \tan^5 \sqrt{x} + C$

5.  $\log(1 + \log x) + C$

6.  $\frac{1}{4a} \cos 2(ax + b) + C$

7.  $\frac{1}{7}(x^3 - 1)^{7/3} + \frac{1}{4}(x^3 - 1)^{\frac{4}{3}} + C$

8.  $\frac{1}{2} \log(e^{2x} + e^{-2x}) + C$

9.  $F(x) = x^4 - 6x + 3$

### ANSWERS KEY SA-II (3 MARKS)

1.  $2\sin\sqrt{x} + C$       2.  $\frac{x}{2} - \frac{1}{2}\log|\cos x + \sin x| + C$   
 3.  $2(\sin x + x \cos x) + C$       4.  $\frac{3}{2\sqrt{2}} \tan^{-1} \sqrt{2} x^2 + C$       5.  $x \log x - x + C$   
 6.  $\frac{1}{2}(x+1) \sqrt{(x^2 + 2x + 5)} + 2\log [(x+1) + \sqrt{(x^2 + 2x + 5)}] + C$   
 7. 0      8.  $\frac{\pi}{4}$       9.  $\frac{1}{(n+1)(n+2)}$   
 10.  $\frac{\pi^2}{32}$       11.  $\frac{19}{99}$

### ANSWER KEY LA (4 MARKS)

1.  $-\frac{1}{3} \cos 3x + \frac{1}{5} \cos 5x + C$       2.  $x \cos a - \sin a \log(x + a) + C$   
 3.  $2\sqrt{\tan x} + C$       4.  $[\frac{\sin 12x}{48} + \frac{x}{4} + \frac{\sin 8x}{32} + \sin \frac{4x}{16}] + C$   
 5.  $\frac{3x}{8} + \frac{1}{8} \sin 4x + \frac{1}{64} \sin 8x + C$       6.  $\sec x - \cosec x + C$   
 7.  $\frac{1}{\sin(a-b)} \log \left| \frac{\cos(x-a)}{\cos(x-b)} \right| + C$   
 8.  $x - 5\log|x-2| + 10\log|x-3| + C$       9.  $-\frac{1}{3} \tan^{-1} x + \frac{2}{3} \tan^{-1} \frac{x}{2} + C$   
 10.  $3 \log(2 - \sin \theta) + \frac{4}{2 - \sin \theta} + C$       11.  $\frac{1}{n} \log \left| \frac{x^n}{x^{n+1}} \right| + c$   
 12.  $\frac{x^2}{2} \sin^{-1} x + \frac{x}{2} \sqrt{(1-x^2)} - \frac{1}{2} \sin^{-1} x + c$   
 13.  $(\sin^{-1} x)^2 x + 2\sqrt{(1-x^2)} \cdot \sin^{-1} x - 2x + c$   
 14.  $\frac{e^x}{(x-1)^2} + C$       15.  $\frac{64}{231}$   
 16.  $\frac{e^2 (e^2 - 2)}{4}$       17.  $\frac{11}{4}$   
 18.  $\frac{\pi \log 2}{2}$       19.  $x \log(\log x) - \frac{x}{\log x} + C$   
 20.  $\sqrt{2} \tan^{-1} \frac{\tan x - 1}{\sqrt{2 \tan x}} + C$       21.  $\frac{\pi^2}{2ab}$   
 22.  $\frac{1}{\sqrt{2}} \tan^{-1} \left[ \frac{\tan x - 1}{\sqrt{2 \tan x}} \right] + \frac{1}{2\sqrt{2}} \log \left[ \frac{\tan x - \sqrt{2 \tan x} + 1}{\tan x + \sqrt{2 \tan x} + 1} \right] + c$   
 23.  $\frac{19}{2}$   
 25.  $\frac{\pi^2 - 4\pi}{16} + \log \sqrt{2}$       26.  $\frac{\tan^{11} x}{11} + \frac{\tan^9 x}{9} + c$   
 27.  $\frac{1}{4} \log|x-1| + \frac{3}{4} \log|x+1| + \frac{1}{2}(x+1)$ .      28.  $\frac{\pi}{4}$   
 29.  $\log \left| \frac{1+\sin x}{2+\sin x} \right| + c$       30.  $\frac{1}{2}(\sin x - \cos x)e^x + c$   
 31.  $\frac{3}{\pi} + 1/\pi^2$   
 32.  $\frac{1}{4} \log|1 + \sin x| + \frac{1}{2(1+\sin x)} - \frac{1}{4} \log|1 - \sin x| + c$   
 33.  $2\log 2$       37.  $\frac{1}{2} \left[ \frac{\sin 2x}{2} + \sin x \right] + C$

### ANSWER KEY CASE STUDY BASED QUESTIONS

1. (i)  $\frac{15}{18}$       (ii). 2  
 2. (i) 0      (ii) 0      (iii) 0  
 3. (i)  $\frac{a}{2}$       (ii) 0

## **CHAPTER: 8 APPLICATION OF INTEGRALS**

### **SHORT ANSWER TYPE QUESTIONS-I (SA-I) (2 MARKS)**

Q.1 Find the area of region bounded by  $y^2 = 4x$ ,  $x = 1$ ,  $x = 4$  and x- axis in the 1<sup>st</sup> quadrant.

Q.2 Find the area of region bounded by the curve  $y = \sin x$  between  $x = 0$  and  $x = \pi$

Q.3 Using integration, find the area of region bounded by curve  $x = 2y + 3$  and the lines  $y = 1$ ,  $y = -1$

Q.4 Find the area of region bounded by  $y^2 - 4x = 0$ , y- axis and  $y = 3$

Q.5 Find the area bounded by curve  $y = x - 3\sqrt{x}$  and x- axis.

Q.6 Find the area enclosed by the curve  $y = -x^2$  and the line  $x + y + 2 = 0$

Q.7 Find the area of parabola  $y^2 = 16x$  bounded by its latus rectum.

Q.8 Find the area under the curve  $y = 2\sqrt{x}$  included between the lines  $x = 0$  and,  $x = 1$

Q.9 Find the area of region bounded by the curves  $y^2 - 9x = 0$ , and  $y = 3x$

Q.10 Find the area of region bounded by curves  $y = x + 1$ , and lines  $x = 2$ ,  $x = 3$

### **SHORT ANSWER TYPE QUESTIONS-II (SA-II) (3 MARKS)**

Q.1 Draw a rough sketch of graph of function  $y = 2\sqrt{1 - x^2}$ ,  $x \in [0,1]$  and evaluate the area between curve and axes.

Q.2 Sketch the graph of  $y = |x + 1|$  and evaluate  $\int_{-3}^1 |x + 1| dx$ .

Q.3 The area between  $x = y^2$  and  $x = 4$  is divided into two equal parts by the line  $x = a$ , find the value of  $a$ .

Q.4 Using integration, find the area of region bounded by line  $y - 1 = x$ , x-axis and the ordinates  $x = -2$ ,  $x = 3$

Q.5 Find the area bounded by the curve  $y = \sin x$  between  $x = 0$ ,  $x = 2\pi$

Q.6 Find the area enclosed by the curves  $x = 3 \cos \theta$  and  $y = 2 \sin \theta$

Q.7 If the area bounded by the parabola  $y^2 = 16a x$  and the line  $y = 4px$  is  $\frac{a^2}{12}$  sq. units, using integration, find the value of  $p$ .

Q.8 Using integration, find the area of region  $\{(x, y) : x^2 \leq y \leq x\}$

Q.9 Find the area of small region by the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and the line  $2x + 3y = 6$

Q.10 Using integration, find the area enclosed by the curve  $y = \frac{3x^2}{4}$  and the line  $3x - 2y = -12$ .

## LONG ANSWER TYPE QUESTIONS (4MARKS)

Q.1 Find the area of following region using integration:

$$\{(x, y): y \leq |x| + 2, y \geq x^2\}$$

Q.2 Find the area of following region using integration:

$$\{(x, y): x^2 + y^2 \leq 1 \leq x + y\}$$

Q.3 Find the area of following region using integration:

$$\{(x, y): x^2 + y^2 \leq 4, y \leq x\sqrt{3}\}$$

Q.4 Using integration, find the area of region in the first quadrant enclosed by the line

$$x + y = 2, \text{ the parabola } y^2 = x \text{ and the x- axis.}$$

## CASE STUDY BASED QUESTIONS

Q.1 An elliptical sized swimming pool is to be constructed in a town whose equation is

$$\frac{x^2}{49} + \frac{y^2}{25} = 1, \text{ then}$$



(i) Find the area of swimming pool as shown in figure using integration.

(ii) If the rate of covering its floor with tiles is Rs 300/m<sup>2</sup>, then find total cost to cover entire floor of swimming pool.

Q.2 A farmer grows paddy crop in a circular plot of land satisfying equation  $x^2 + y^2 = 10000$



(i) Find the area of circular plot growing paddy plants by using integration is. ( $\pi=3.14$ )

(ii) If rate of expense for production of crop per sq.m is Rs 20, find the total expense.

Q.3 For a Triangular natural lake, the equations of whose three sides are: -

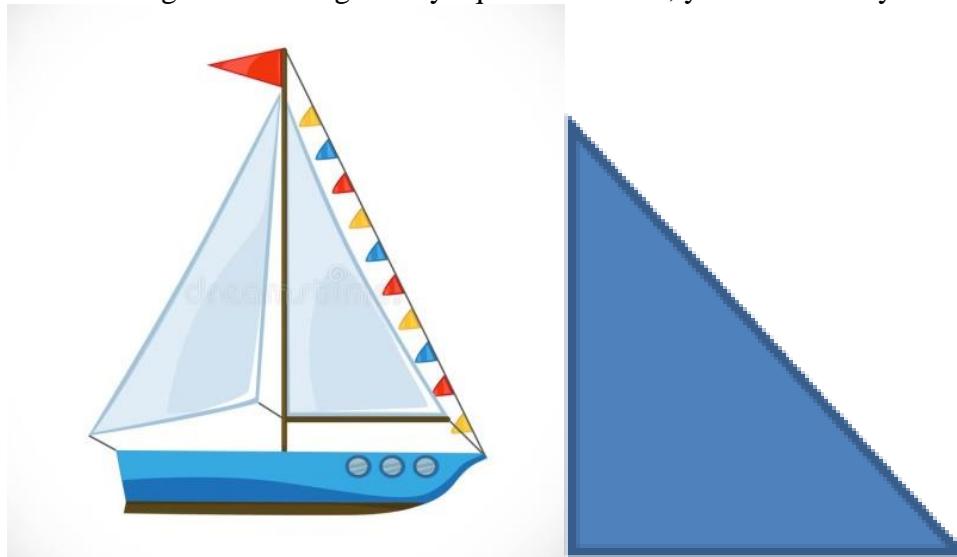
$2x+y=4$ ,  $3x-2y=6$  and  $x-3y+5=0$  as given in figure below



**a alamy stock photo**

- Find the area of triangular lake using integration as shown in figure.
- Find the height of the lake on largest side.

Q.4 Now a days, almost every boat has a triangular sail. By using a triangular sail design, it has become possible to travel against the wind using a technique known as tacking. Tacking allows the boat to travel forward with the wind at right angles to the boat. A student designs a boat with triangular sail on the walls and three edges (lines) at the triangular sail are given by equations:  $x = 0$ ,  $y = 0$  and  $2x + y - 4 = 0$  respectively.



On the basis of above information, answer the following question.

- Find the point of intersection of the edges (lines)
  - $2x + y - 4 = 0$  and  $y = 0$
  - $2x + y - 4 = 0$  and  $x = 0$
- Find the area of the triangular sail using integration.

### ANSWER KEY SA-I (2 MARKS)

1.  $\frac{28}{3}$  sq. units      2. 2sq. units      3. 6 sq. units

4.  $\frac{9}{4}$  sq. units

5.  $\frac{27}{2}$  sq. units Curve meets x- axis, then  $y = 0$   
Therefore,  $x - 3\sqrt{x} = 0$ , getting  $x = 0, 9$   
Area =  $\int_0^9 (x - 3\sqrt{x}) dx = \frac{27}{2}$

6.  $\frac{9}{2}$  sq. units      7.  $\frac{128}{3}$  sq. units      8.  $\frac{4}{3}$  sq. units

9.  $\frac{1}{2}$  sq. units      10.  $\frac{7}{2}$  sq. units

### ANSWER KEY SA-II (3 MARKS)

1.  $\frac{\pi}{2}$  sq. units      2. 4 sq. units.      3.  $\sqrt[3]{16}$

4. Area =  $\int_{-2}^{-1} -(x + 1) dx + \int_{-1}^3 (x + 1) dx = \frac{17}{2}$       5. 4 sq. units

6.  $6\pi$  sq. units  
 $x = 3 \cos \theta$  and  $y = 2 \sin \theta$   
Squaring and adding, we get ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$   
Required area =  $6\pi$  sq. units

7.  $2\sqrt{2}$  sq. units      8.  $\frac{1}{6}$  sq. units      9.  $\frac{3(\pi-2)}{2}$

10. 27 sq. units

### ANSWER KEY LA (4 MARKS)

1.  $\frac{20}{3}$  sq. units.      2.  $\left(\frac{\pi}{4} - \frac{1}{2}\right)$  sq. units      3.  $\frac{2\pi}{3}$  sq. units

4.  $\frac{7}{6}$  sq. units.

### CASE STUDY BASED QUESTIONS ANSWER KEY

1. (i)  $110 \text{ m}^2$       (ii) Rs. 33000

2. (i)  $31400 \text{ m}^2$       (ii) Rs. 628000

3. (i) 3.5 sq. km      (ii)  $\frac{7}{\sqrt{13}}$  km

4. (a) (2, 0) and (0, 4)      (b) 4 sq. units.

## **CHAPTER: 9 DIFFERENTIAL EQUATIONS**

### **SHORT ANSWER TYPE QUESTIONS-I (SA-I) (2 MARKS)**

Q.1 Find the general solution of the differential equation  $\frac{ydx - xdy}{y} = 0$ .

Q.2 Find the sum of order and degree of differential equation  $(y'')^2 + (y''')^3 + (y')^4 + y^5 = 0$ .

Q.3 Write the degree of the differential equation  $x \left( \frac{d^2y}{dx^2} \right)^3 + y \left( \frac{dy}{dx} \right)^4 + x^3 = 0$ .

Q.4 Find the integrating factor of the differential equation  $\frac{dy}{dx} + xy = x^2$ .

Q.5 Write the sum of the order and degree of the differential equation:  $\frac{d}{dx} \left[ \left( \frac{d^2y}{dx^2} \right)^4 \right] = 0$ .

Q.6 Write the integrating factor of the differential equation:  $(1 + x^2) \frac{dy}{dx} + y = \tan^{-1} x$

Q.7 Write solution of the differential equation:  $\frac{dy}{dx} = e^x + 2x$ .

Q.8 Find the degree of  $\frac{dy}{dx} + \sin \left( \frac{dy}{dx} \right) = 0$

Q.9 Write the order and degree of  $\frac{d^2y}{dx^2} = 1 + \sqrt{\frac{dy}{dx}}$

Q.10 Solve,  $\log \left( \frac{dy}{dx} \right) = ax + by$ .

Q.11 Find the particular solution of the differential equation  $\frac{dy}{dx} = y \tan x$ ,  $y=1$  when  $x=0$ .

Q.12 Find the general solution of  $\frac{dy}{dx} + 2 \tan x y = \sin x$ .

Q.13 Find the integrating factor of Differential equation  $(1 + y^2) + (x - \tan^{-1} y) \frac{dy}{dx} = 0$

Q.14 Solve  $\frac{dy}{dx} = \frac{y}{x} + \log x$ .

Q.15 Show that  $y = 2(x^2 - 1) + c e^{-x^2}$  is the solution of differential equation  $\frac{dy}{dx} + 2xy = 4x^3$

Q.16 Solve the differential equation  $\frac{dy}{dx} = \frac{x e^x \log x + e^x}{x \cos y}$ .

Q.17 Prove that  $xdy - ydx = \sqrt{x^2 + y^2} dy$  is a homogeneous differential equation

Q.18 Verify that whether  $[y - x \cos \frac{dy}{dx}] dy + [y \cos \frac{y}{x} - 2x \sin \frac{y}{x}] dx = 0$  is homogeneous differential equation or not?

Q.19 Solve  $(1 + y^2)(1 + \log x) dx + x dy = 0$ .

Q.20 Solve  $\frac{dy}{dx} = 1 + x + y + xy$ .

Q.21 Solve,  $e^x \sqrt{1 - y^2} dx + \frac{y}{x} dy = 0$ .

Q.22 Solve  $\frac{dy}{dx} + \frac{2x}{x^2-1} y = \frac{2}{(x^2-1)^2}$ .

Q.23 Solve  $(x - y^3) dy + y dx = 0$ .

### SHORT ANSWER TYPE QUESTIONS-II (SA-II) (3 MARKS)

Q 1. Solve the differential equation  $y' + y = e^x$ .

Q 2. Find the particular solution of the differential equation  $\frac{dy}{dx} = -4xy^2$  given that  $y = 1$ , when  $x = 0$ .

Q 3. Find the particular solution of the differential equation:

$$x(1 + y^2) dx - y(1 + x^2) dy = 0, \text{ given that } y = 1 \text{ when } x = 0.$$

Q 4. Solve:  $ydx - xdy = x^2ydx$ .

Q 5. Solve:  $\sec^2 x \tan \tan y dx + \sec^2 y \tan \tan x dy = 0$ .

Q 6. Solve the differential equation  $\frac{dy}{dx} = e^{x+y} + e^{-x+y}$ .

Q 7. Solve the differential equation:  $\frac{dy}{dx} = \frac{y-x}{y+x}$ .

Q 8. Solve:  $\frac{dy}{dx} = \frac{x-y \cdot \cos \cos x}{1 + \sin x}$ .

Q 9. Solve the differential equation  $\frac{dx}{dy} = \frac{x}{y} - \operatorname{cosec} \frac{x}{y}$  given that  $y(0) = 1$ . [Hint. Put  $x = vy$ ]

Q 10. Show that the solution of differential equation  $\frac{dy}{dx} = \frac{x+2y}{x}$  is  $x + y = kx^2$ . Where  $k$  is a constant.

Q 11. Solve:  $\frac{dy}{dx} + y \sec x = \tan x \left(0 \leq x < \frac{\pi}{2}\right)$

Q 12. Solve the following differential equation:

$$\left[y - x \cos\left(\frac{y}{x}\right)\right] dy + \left[y \cos\left(\frac{y}{x}\right) - 2x \sin\left(\frac{y}{x}\right)\right] dx = 0$$

Q 13. Find a particular solution of the differential equation  $(x - y)(dx + dy) = dx - dy$ , given that  $y = -1$ , when  $x = 0$ .

Q 14. Solve the following differential equation, given that  $y = 0$ , when  $x = \frac{\pi}{4}$ :

$$\sin 2x \frac{dx}{dy} = \tan x + y.$$

Q.15. Find the solution of differential equation  $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ .

Q.16. Solve  $(x+y)^2 \frac{dy}{dx} = a^2$

Q.17. Show that the differential equation  $2xy \frac{dy}{dx} = x^2 + 3y^2$  is homogeneous and solve it.

### LONG ANSWER TYPE QUESTIONS (4 MARKS)

Q.1 Solve the differential equation:  $xdy - ydx = \sqrt{x^2 + y^2} dx$ .

Q.2 Solve the differential equation:  $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$

Q.3 Solve the differential equation  $(1 + e^{2x}) dy + (1 + y^2)e^x dx = 0$  given that when  $x=0$  and  $y=1$ .

Q.4 Find the particular solution of the differential equation  $\frac{dy}{dx} = 1 + x + y + xy$ , Given that  $y = 0$  when  $x = 1$ .

Q.5 Solve the differential equation  $\frac{dx}{dy} + x \cot y = 2y + y^2 \cot y$

Q.6 Solve:  $y\{x \cos \frac{y}{x} + y \sin \frac{y}{x}\}dx - x\{y \sin \frac{y}{x} - x \cos \frac{y}{x}\}dy = 0$

Q.7 Find the particular solution of the differential equation satisfying the given conditions:  $(1 + x^2) \frac{dy}{dx} + 2xy = \frac{1}{(1+x^2)}$ ;  $y=0$  when  $x = 0$ .

Q.8 Show that the differential equation  $2xy \frac{dy}{dx} = x^2 + 3y^2$  is homogeneous and solve it.

Q.9 Solve the differential equation:  $\left\{ x \cos \left( \frac{y}{x} \right) + y \sin \frac{y}{x} \right\} ydx = \left\{ y \sin \frac{y}{x} - x \cos \frac{y}{x} \right\} xdy$

Q.10 Solve the following differential equation:  $(1 + y^2) dx = (\tan^{-1} y - x) dy$

Q.11 Solve the differential equation:  $(1+x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$

Q.12 Solve the differential equation:  $\frac{dy}{dx} + 2y \tan x = \sin x$ , given that  $y = 0$ , when  $x = \frac{\pi}{3}$ ,

Q.13 Solve:  $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$ , given  $y = 0$  when  $x = \frac{\pi}{2}$ .

Q.14 Find the particular solution of the differential equation:  $(1 + x^2) \frac{dy}{dx} = (e^{m \tan^{-1} x} - y)$  given that  $y = 1$ , when  $x = 0$

Q.15 Solve the following differential equation, given that  $y = 0$ , when  $x = \frac{\pi}{4}$

:  $\sin 2x \frac{dy}{dx} - y = \tan x$

## CASE STUDY BASED QUESTIONS

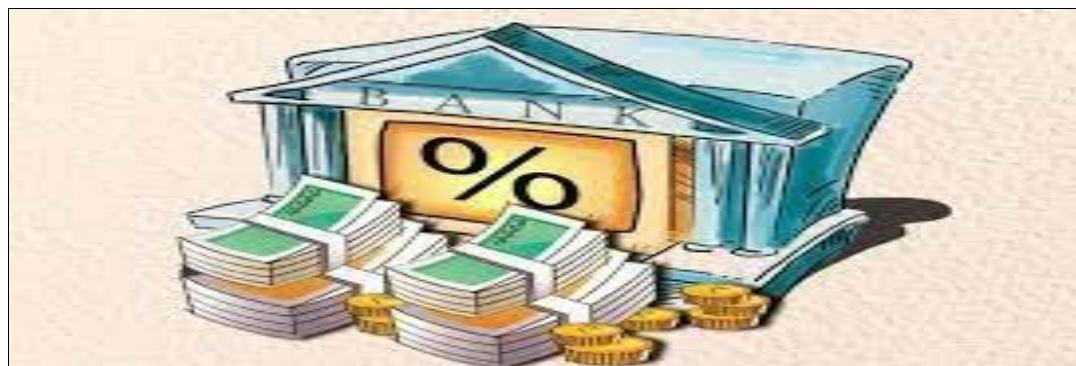
Q.1 The rumour in whatsapp spread in a population of 10000 people at a rate proportional to the Product of the number of people who have heard it and the number of who have not. Also, it is given that 200 people initiate the rumour and the total of 1000 people know the rumour after 2 days



Based on the above information answer the following questions

- (i) If  $y(t)$  denotes the number of people who know the rumour at the instant  $t$  then maximum value of  $y(t)$  is
  - (a) 500
  - (b) 1000
  - (c) 5000
  - (d) 10000
- (ii)  $\frac{dy}{dt}$  is proportional to
  - (a)  $(y- 10000)$
  - (b)  $y(y- 1000)$
  - (c)  $y(1000-y)$
  - (d)  $y(10000-y)$
- (iii) The value of  $y(0)$  is
  - (a) 500
  - (b) 100
  - (c) 200
  - (d) 1000
- (iv) The value of  $y(0)$  is
  - (a) 500
  - (b) 1000
  - (c) 2000
  - (d) 10000

Q.2 It is known that if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of the bank interest per annum and the principal. Let  $P$  denotes the principal at any time  $t$  and rate of interest  $r\%$  per annum.



Based on the above information answer the following questions

- (i) Find the value of  $\frac{dp}{dt}$ 
  - (a)  $\frac{pr}{1000}$
  - (b)  $\frac{pr}{100}$
  - (c)  $\frac{pr}{10}$
  - (d)  $pr$

(ii) If  $P_0$  be the initial principal. Then find the solution of differential equation formed by the given situation  
 (a)  $\text{Log}(\frac{P}{P_0}) = \frac{rt}{100}$    (b)  $\text{Log}(\frac{P}{P_0}) = \frac{rt}{10}$    (c)  $\text{Log}(\frac{P}{P_0}) = rt$  (d)  $\text{Log}(\frac{P}{P_0}) = 100rt$

(iii) If the interest is compound continuously at 5% per annum. In how many years Rs 100 is double itself.  
 (a) 12.728 yrs   (b) 14.789 yrs   (c) 13.862 yrs   (d) 15.872 yrs

(iv) At what interest rate will Rs 100 double after 10 Years ( $\log 2 = 0.6932$ )  
 (a) 9.66%   (b) 8.239%   (c) 7.341%   (d) 6.931%

(v) How much will Rs 1000 worth at 5% interest after 10 years ( $e^{0.5} = 1.648$ )  
 (a) Rs 1648   (b) Rs 1500   (c) Rs 1664   (d) Rs 1572

Q.3 Polio drops are delivered to 20K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation  $dy/dx = (20 - y)$  where x denotes the number of weeks and y the number of children who have been given the drops.



(i) State the order of the above given differential equation.

(ii) Which method of solving a differential equation can be used to solve  $dy/dx = (20 - y)$ .  
 a. Variable separable method   b. Solving Homogeneous differential equation  
 c. Solving Linear differential equation   d. all of the above

(iii) Find the solution of the differential equation  $dy/dx = (20 - y)$ .

(iv) Find the value of c in the particular solution given that  $y(0) = 0$  and  $k = 0.049$ .

(v) Which of the following solutions may be used to find the number of children who have been given the polio drops?  
 a.  $y = 20 - e^{kx}$    b.  $y = 20 - e^{-kx}$    c.  $y = 20 (1 - e^{-kx})$    d.  $y = 20 (e^{-kx} - 1)$

## ANSWER KEY SA-I (2 MARKS)

1.	$y = cx$
2.	Order = 3 and degree = 3, So sum=6
3.	Degree = 2
4.	$I.F. = e^{\int x dx} = e^{\frac{x^2}{2}}$
5.	4
6.	$IF = e^{\tan^{-1} x}$
7.	$y = e^x + x^2 + c$
8.	Not defined
9.	$\frac{d^2y}{dx^2} = 1 + \sqrt{\frac{dy}{dx}}, \Rightarrow \left(\frac{d^2y}{dx^2} - 1\right)^2 = \frac{dy}{dx}$ Now, Order= 2 and Degree=2
10.	$\frac{dy}{dx} = e^{ax+by}$ Ans : $\frac{-1}{b} e^{-by} = \frac{1}{a} e^{ax} + c$
11.	$\log y = \log \sec x + \log C$ $C = 1$ $Y = \sec x$
12.	$IF = \sec^2 x$ Sol. is $y \cdot \sec^2 x = \sec x + c$
13.	Writing $\frac{dy}{dx} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1} y}}{1+y^2}$ $IF = e^{\tan^{-1} y}$
14.	$IF = \frac{1}{x}$ $\frac{y}{x} = \log(\log x) + C$
15.	$\frac{dy}{dx} = 4x - 2xc e^{-x^2}$
16.	$\sin y + e^x \log x + C$
17.	$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$ Prove that $f(kx, ky) = k^0 f(x, y)$
18.	Writing $\frac{dy}{dx} =$ required function, Prove that $f(kx, ky) = k^0 f(x, y)$
19.	$\log x + \frac{(\log x)^2}{2} + \tan^{-1} y + C = 0$
20.	$\log(1+y) = x + \frac{x^2}{2} + C$
21.	$x e^x - e^x - \sqrt{1 - y^2} = C$
22.	If $x^2 - 1$ General sol is $y(x^2 - 1) = \log(\frac{x-1}{x+1}) + C$
23.	$IF = y$ Solution is $xy = \frac{y^4}{4} + C$

### ANSWER KEY SA-II (3 MARKS)

1.  $y = \frac{e^x}{2} + C e^{-x}$
2.  $C = -1, (2x^2 + 1).y = 1$
3.  $y = \sqrt{2x^2 + 1}$
4.  $y = C \cdot x \cdot e^{\frac{-x^2}{2}}$
5.  $xy = C$
6.  $e^{-y} + e^x - e^{-x} + C = 0$
7.  $\tan^{-1} \frac{y}{x} + \frac{1}{2} \log \log \left(1 + \left(\frac{y}{x}\right)^2\right) = \log(Cx)$
8.  $\frac{y}{1+\sin x} = \frac{x^2}{2} + C$
9.  $C = -1, \cos\left(\frac{x}{y}\right) = \log \log y + 1$
10. Show that the solution of differential equation  $\frac{dy}{dx} = \frac{x+2y}{x}$  is  $x+y = kx^2$ . Where k is a constant.
11.  $y(\sec x + \tan x) = \sec x + \tan x - x + C$
12.  $y^2 - 2x^2 \cos\left(\frac{y}{x}\right) = C$
13.  $\log \log |x - y| = x + y + 1$
14. I.F.  $= \frac{1}{\sqrt{\tan \tan x}}$ ,  $C = -1$  and solution is  $y = \tan \tan x - \sqrt{\tan \tan x}$
15.  $y(1+x^2) = \tan^{-1} x + C$
16.  $y = a \tan^{-1}\left(\frac{x+y}{a}\right) - C$
17.  $x^2 + y^2 = cx^3$

### ANSWER KEY LA (4 MARKS)

1.  $\{y + \sqrt{x^2 + y^2}\}^2 = C^2 x^4$
2.  $\log |y| + \frac{1}{y} = -\frac{1}{x} + x + C$
3.  $\tan^{-1} y = \tan^{-1} \frac{1}{e^x} \Rightarrow y = \frac{1}{e^x}$
4.  $\log|1+y| = x + \frac{x^2}{2} - \frac{3}{2}$
5.  $x = y^2 - \frac{\pi^2}{4} \cosec y$
6.  $|xy \cos\left(\frac{y}{x}\right)| = k$ ,  $x \neq 0$ ,  $k > 0$
7. GS:  $y(1+x^2) = \tan^{-1} x + C$  and PS:  $y = \frac{\tan^{-1} x}{1+x^2}$
8.  $x^2 + y^2 = cx^3$
9.  $xy \cos \frac{y}{x} = A$
10.  $xe^{\tan^{-1} y} = \tan^{-1} y e^{\tan^{-1} y} - e^{\tan^{-1} y} + C$
11.  $y = \frac{1}{2} e^{\tan^{-1} x} + ce^{-\tan^{-1} x}$
12.  $y = \cos x - 2\cos^2 x$
13.  $y \sin x = 2x^2 - \frac{\pi^2}{2}$
14.  $y \cdot e^{\tan^{-1} x} = \frac{e^{(m+1)\tan^{-1} x}}{m+1} + \frac{m}{m+1}$
15.  $y = \tan x - \sqrt{\tan x}$

### CASE STUDY BASED QUESTIONS ANSWER KEY

1. (i) (d) 10000 (ii) (d)  $y(10000-y)$  (iii) (c) 200 (iv) (b) 1000
2. (i) (b)  $\frac{pr}{100}$  (ii) (a)  $\log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$  (iii) (c) 13.862 Yrs  
(iv) (d) 6.931%
3. (i) Order is 1 (ii) (a) Variable separable method (iii)  $-\log|20-y| = kx + C$   
(iv)  $\log 1/20$  (v)  $y = 20(1 - e^{-kx})$

## **CHAPTER: 10 VECTOR ALGEBRA**

### **SHORT ANSWER TYPE QUESTIONS-I (SA-I) (2 MARKS)**

Q1. If  $\vec{a}$  and  $\vec{b}$  are unit vectors then prove that  $|\hat{a} + \hat{b}| = 2 \cos \frac{\theta}{2}$ , where  $\theta$  is the angle between them.

Q2. If a unit vector  $\vec{a}$  makes angles  $\frac{\pi}{3}$  with  $\hat{i}$ ,  $\frac{\pi}{4}$  with  $\hat{j}$  and an angle  $\theta$  with  $\hat{k}$ , then find the value of  $\theta$ .

Q3. Find the angle between two vectors  $\vec{a}$  and  $\vec{b}$  having the same length  $\sqrt{2}$  and their vector product is  $-\hat{i} - \hat{j} + \hat{k}$ .

Q4. Find the magnitude of each of the two vectors  $\vec{a}$  and  $\vec{b}$ , having the same magnitude such that the angle between them is  $60^\circ$  and their scalar product is  $\frac{9}{2}$ .

Q5. If  $\theta$  is the angle between two vectors  $\hat{i} - 2\hat{j} + 3\hat{k}$  and  $3\hat{i} - 2\hat{j} + \hat{k}$ , find  $\sin \theta$ .

Q6. Find the area of parallelogram, whose adjacent sides are determined by the vectors  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$  and  $\vec{b} = 2\hat{i} + \hat{j} - 4\hat{k}$ .

Q7. Find a unit vector in the direction of  $(\vec{a} + \vec{b})$  where  $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ .

Q8. Show that the points A, B, C with position vectors  $2\hat{i} - \hat{j} + \hat{k}$ ,  $\hat{i} - 3\hat{j} - 5\hat{k}$  and  $3\hat{i} - 4\hat{j} - 4\hat{k}$  respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

Q9. If  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ , then show that the angle  $\theta$  between  $\vec{b}$  and  $\vec{c}$  is given by

$$\cos \theta = \frac{|\vec{a}|^2 - |\vec{b}|^2 - |\vec{c}|^2}{2|\vec{b}||\vec{c}|}.$$

Q10. If  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are three vectors such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$  and  $|\vec{a}| = 3$ ,  $|\vec{b}| = 5$  and  $|\vec{c}| = 7$  then find the angle between  $\vec{a}$  and  $\vec{b}$ .

### **SHORT ANSWER TYPE QUESTIONS-II (SA-II) (3 MARKS)**

Q1. If  $\vec{a} \neq \vec{0}$ ,  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ ,  $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ , then show that  $\vec{b} = \vec{c}$ .

Q2. The scalar product of the vector  $\hat{i} + \hat{j} + \hat{k}$  with a unit vector along the sum of vectors  $2\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$  is equal to one. Find the value of  $\lambda$ .

Q3. Show that the points with position vectors  $2\hat{i} + \hat{j} - \hat{k}$ ,  $3\hat{i} - 2\hat{j} + \hat{k}$  and  $\hat{i} + 4\hat{j} - 3\hat{k}$  are collinear. Hint:  $\vec{AB} \times \vec{BC} = \vec{0}$

Q4. Let  $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$  and  $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ . Find a vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$  and  $\vec{b}$  and  $\vec{c} \cdot \vec{d} = 27$ .

Q5. If  $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j}$  are such that  $\vec{a} + \lambda\vec{b}$  is perpendicular to  $\vec{c}$  then find the value of  $\lambda$ .

Q6. Find the area of a parallelogram whose diagonals are determined by the vectors  $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$  and  $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$

Q7. If  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = \hat{j} - \hat{k}$ , find a vector  $\vec{c}$  such that  $\vec{a} \times \vec{c} = \vec{b}$  and  $\vec{a} \cdot \vec{c} = 3$

Q8. The magnitude of the vector product of the vector  $\hat{i} + \hat{j} + \hat{k}$  with a unit vector along the sum of vector  $2\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$  is equal to  $\sqrt{2}$ . Find value of  $\lambda$ .

Q9. Find a unit vector perpendicular to each of the vectors  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ . where  $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ .

### CASE STUDY BASED QUESTIONS

Q.1 A lighthouse is a tower, building, or other type of structure designed to emit light from a system of lamps and lenses and to serve as a beacon for navigational aid, for maritime pilots at sea or on inland waterways.

Four light houses are located at different at locations to guide the ships in the sea. The location of their top are given by A (5,9,5), B (19,9,5), C (19,16,9) and D (5,16,9). Answer the following questions based on the above information:

- (i) Write the  $\vec{BC}$  in standard form.
- (ii) Find the magnitude of  $\vec{CD}$ .
- (iii) Write the components of  $\vec{N}$  perpendicular to both  $\vec{CD}$  and  $\vec{AD}$ .
- (iv) What are the components of  $\vec{AB}$ .
- (v) Which two vectors have the same magnitude?

**Q2** A quantity that has magnitude as well as direction is called a vector. Unit vectors are vectors whose magnitude is exactly 1 unit. The unit vector in the direction of a given vector  $\vec{a}$  is denoted by  $\hat{a}$ . The unit vectors along the axes are denoted by  $\hat{i}$ ,  $\hat{j}$  and  $\hat{k}$  respectively.

Answer the following questions:

- (i) Find the value of  $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$
- (ii) If  $\vec{a}$  is a non-zero vector of magnitude  $a$ , and  $\lambda$  a non-zero scalar, then  $\lambda \vec{a}$  is unit vector if (a)  $\lambda = 1$  (b)  $\lambda = -1$  (c)  $a = |\lambda|$  (d)  $a = 1/|\lambda|$
- (iii) Find the vector in the direction of the vector  $\hat{i} - 2\hat{j} + 2\hat{k}$ , that has magnitude 9.
- (iv) Find the vectors having initial and terminal points as  $(-2, 6, 1)$  and  $(-3, 5, 3)$  respectively.
- (v) For any vector  $\vec{a}$ , find the value of  $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ .

### ANSWER KEY SA-I (2 MARKS)

1. Prove	2. $\frac{\pi}{3}$	3. $\frac{\pi}{3}$	4. $ \vec{a}  =  \vec{b}  = 3$
$\frac{2\sqrt{6}}{7}$		$\frac{1}{3}(\hat{i} + 2\hat{j} - 2\hat{k})$	
5.	6. $5\sqrt{6}$ sq. unit.	7.	
8. $\frac{1}{2}\sqrt{210}$	9. Show	10. $\theta = 600$	

### ANSWER KEY SA-II (3 MARKS)

1. Show	2. $\lambda = 1$	3. Show
4. $\vec{d} = 96\hat{i} - 3\hat{j} - 42\hat{k}$	5. $\lambda = 8$	6. $\frac{1}{2} \vec{a} \times \vec{b}  = 5\sqrt{3}$ sq. unit.
7. $\vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$ .	8. $\lambda = 1$	9. $\hat{n} = \pm \frac{1}{3}(2\hat{i} - 2\hat{j} - \hat{k})$

### CASE STUDY BASED QUESTIONS ANSWER KEY

1 Solutions:

- (i)  $\vec{BC} = (19 - 19)\hat{i} + (16 - 9)\hat{j} + (9 - 5)\hat{k} = 7\hat{j} + 4\hat{k}$
- (ii)  $|\vec{CD}| = \sqrt{(5 - 19)^2 + (16 - 16)^2 + (9 - 9)^2} \text{ units} = 14 \text{ units}$
- (iii)  $\vec{N} = \vec{CD} \times \vec{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 14 & 0 & 0 \\ 0 & 7 & 4 \end{vmatrix} = -56\hat{j} + 98\hat{k}$
- (iv)  $\vec{AB} = (19 - 5)\hat{i} + (9 - 9)\hat{j} + (5 - 5)\hat{k} = 14\hat{i}$ ; Components are 14, 0, 0
- (v)  $|\vec{AD}| = |\vec{BC}| = \sqrt{65} \text{ units}$

2. (i)  $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j}) = \hat{i} \cdot \hat{i} + \hat{j} \cdot (-\hat{j}) + \hat{k} \cdot \hat{k} = 1$

(ii)  $\alpha = 1/|\lambda|$

(iii) Unit vector in the direction of the vector  $\hat{i} - 2\hat{j} + 2\hat{k}$  is  $= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$   
So required vector  $= 3(\hat{i} - 2\hat{j} + 2\hat{k})$

(iv)  $-\hat{i} - \hat{j} + 2\hat{k}$

(v) Let  $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ , then  $\vec{a} \times \hat{i} = z\hat{j} - y\hat{k}$   
So,  $(\vec{a} \times \hat{i})^2 = y^2 + z^2$   
 $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2 = 2\vec{a}^2$

## **CHAPTER: 11 THREE-DIMENSIONAL GEOMETRY**

### **SHORT ANSWER TYPE QUESTIONS-I (SA-I) (2 MARKS)**

Q1. Show that the lines  $\frac{x-5}{7} = \frac{y+2}{-5} = z$  and  $x = \frac{y}{2} = \frac{z}{3}$  are perpendicular to each other.

Hint:  $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Q2. The equation of a line is  $5x - 3 = 15y + 7 = 3 - 10z$  write the direction cosines of the line.

Q3. Find the value of  $\lambda$  so that the lines  $\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$  and  $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$  are perpendicular to each other.

Q4. Find the vector and cartesian equation of the line that passes through the points (3, -2, -5) and (3, -2, 6)

Q5. Find the distance between two planes  $2x - y + 2z = 5$  and  $5x - 2.5y + 5z = 20$ .

Q6. Find the distance of a point (2,3,4) from the plane  $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = -11$ .

Q7. Write the equation of a plane which is at a distance of  $5\sqrt{3}$  units from the origin and the normal to which is equally inclined to the coordinate axes.

Q8. Write the direction cosines of the normal to the plane  $3x + 4y + 12z = 52$

Q9. Find the equation of the line through the point (1,-1,1) and perpendicular to the lines joining the points (4,3,2), (1,-1,0) and (1,2,-1), (2,1,1).

Q10. Find the image of the point P(3,5,3) in the line  $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ .

### **SHORT ANSWER TYPE QUESTIONS-II (SA-II) (3 MARKS)**

Q1. Find the shortest distance between the following pair of lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

Q2. Find the coordinates of the foot of the perpendicular and perpendicular distance of the point (1,3,4) from the plane  $2x - y + z + 3 = 0$ .

Q3. Find the equation of the plane which contains the intersection of the planes  $\vec{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) - 4 = 0$  and  $\vec{r} \cdot (-2\hat{i} + \hat{j} + \hat{k}) + 5 = 0$  and whose intercept on x-axis is equal to that of on y-axis.

Q4 Find the equation of the plane passing through the point (1,2,1) and is perpendicular to the line joining the points (1,4,2) and (2,3,5). Also find the perpendicular distance of the plane from the origin.

Q5 Find whether the lines  $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} + \hat{j})$  and  $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$  intersect or not. If intersecting, find their point of intersection.

Q6. Find the coordinates of the point where the line through the points (3,4,1) and (5,1,6) crosses the XY- plane.

Q7 Find the distance of the point (2,3,4) from the line  $\frac{x+3}{3} = \frac{y-3}{6} = \frac{z}{2}$  measured parallel to the plane  $3x+2y+2z-5=0$

Q8 Find the equation of the plane through the point (-4,-3,2) and perpendicular to the line of intersection of the planes  $x-2y+2z-3=0$  and  $2x-y-3z=0$ . Also find the point of intersection of the line  $\vec{r} = (\hat{i} - 2\hat{j} - \hat{k}) + \lambda(\hat{i} + 3\hat{j} - 9\hat{k})$  and the plane obtained above.

### LONG ANSWER TYPE QUESTIONS (4 MARKS)

Q.1 Find the shortest distance between the lines  
 $\vec{r} = (4\hat{i} - \hat{j}) + \lambda((\hat{i} + 2\hat{j} - 3\hat{k})$  and  $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu((2\hat{i} + 4\hat{j} - 5\hat{k})$

Q.2 Find the equation of the plane which contains the line of intersection of the planes  $x+2y+3z-4=0$  and  $2x+y-z+5=0$  and whose x -intercept is twice its z intercept.

Q.3 Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point P(5,4,2) to the line  $\vec{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda((2\hat{i} + 3\hat{j} - \hat{k})$ .

Q.4 Find the distance of the point P(3,4,4) from the point ,where the line joining the points A(3,-4,-5) and B(2,-3,1) intersects the plane  $2x+y+z=7$ .

Q.5 Find the distance of the point (1,-2,3) from the plane  $x-y+z=5$  measured parallel to the line whose direction cosines are proportional to 2,3,-6

### CASE STUDY BASED QUESTIONS

Q.1 While climbing up a hill, a person moves along a straight path denoted by:

$$l: \frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$$

With reference to the line  $\mathbf{l}$ , answer the following questions:

- (i) Find the vector equation of given line.
- (ii) Find the unit vector in the direction of vector parallel to the given line.
- (iii) If y-coordinate of a point on this line is 14, then find the x -coordinate of that point.
- (iv) Find the direction ratio of the line.

Q.2 In a classroom, a projector is hanging from the ceiling. Two LED bulbs and a fan is also hanging from the ceiling. If their coordinates are as follows:

Projector: (3, 4, 2)

LED Bulb 1: (2, 3, 2)

LED Bulb 2: (2, 2, 1)

Fan: (3, 4, 1)

Answer the following questions on the basis of above information:

- (i) Find the equation of the plane passing through the LED Bulbs and Fan.
- (ii) Find the height of projector from the plane passing through the LED Bulbs and Fan .
- (iii) Find the equation of perpendicular drawn from projector to the plane passing through the LED Bulbs and Fan.
- (iv) Find the vector equation of the plane passing through the LED Bulbs and Fan.
- (v) Find the coordinates of foot of perpendicular drawn from projector to the plane passing through the LED Bulbs and Fan.

Q.3 There are two planes  $P_1: x + 2y + 3z - 4 = 0$  and  $P_2: 2x + y - z + 5 = 0$ .  $P_3$  is a plane passing through the intersection of  $P_1$  and  $P_2$ , and  $P_3$  is also perpendicular to plane  $P_4: 5x + 3y + 6z + 8 = 0$

Answer the questions:

- (i) Find the equation of plane  $P_3$ .
- (ii) Find the equation of plane parallel to plane  $P_4$  and passing through (2, 3, -4).
- (iii) Which point lies on the plane  $P_1$ ?
- (iv) What is the distance of  $P_2$  from the point (-1, 1, 4)?
- (v) Find the equation of plane parallel to  $P_2$  and which is at a distance of 2 units from the origin is

### ANSWER KEY SA-I (2 MARKS)

1. Show 2.  $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$  3.  $\lambda = -2$   
 4.  $\vec{r} = (3\hat{i} - 2\hat{j} - 5\hat{k}) + (0\hat{i} + 0\hat{j} + 11\hat{k})$ ,  $\frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11}$   
 5.  $d=1$  6. 1 7.  $x + y + z = 15$   
 8.  $\left\langle \frac{3}{13}, \frac{4}{13}, \frac{12}{13} \right\rangle$  9.  $\vec{r} = (\hat{i} - \hat{j} + k) + \lambda(10\hat{i} - 4\hat{j} - 7\hat{k})$   
 10.  $(-1, 1, 7)$

### ANSWER KEY SA-II (3 MARKS)

1.  $d = \frac{1}{\sqrt{6}}$  2.  $(-1, 4, 3)$ ,  $d = \sqrt{6}$  3.  $x + y - 4z = 1$   
 4.  $x - y + 3z - 2 = 0$ ,  $d = \frac{2\sqrt{11}}{11}$  5.  $(3, 0, -1)$  6.  $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$   
 7.  $d = \sqrt{33}$  8.  $[\vec{r} - (4\hat{i} - 3\hat{j} + 2\hat{k})] \cdot (5\hat{i} + 7\hat{j} + \hat{k}) = 0$  and point of intersection  $(0, -1, 8)$

### ANSWER KEY LA (4 MARKS)

1.  $\frac{6}{\sqrt{5}}$  2.  $7x + 11y + 14z = 15$  3.  $(1, 6, 0)$ ,  $2\sqrt{6}$  units  
 4. 7 units 5. 1 unit

### CASE STUDY BASED QUESTIONS

1. (i)  $\vec{r} = -3\hat{i} + 4\hat{j} - 8\hat{k} + \mu(3\hat{i} + 5\hat{j} + 6\hat{k})$   
 (ii)  $\frac{3\hat{i} + 5\hat{j} + 6\hat{k}}{\sqrt{70}}$   
 (iii) Random point on the line is  $(3\alpha - 3, 5\alpha + 4, 6\alpha - 8)$   
 $5\alpha + 4 = 14$ ,  $\alpha = 2$   
 So, x-coordinate is 3  
 (iv) Direction Ratio: 3, 5, 6

2. (i)  $2x - y + z = 3$   
 (ii)  $\frac{1}{\sqrt{6}}$  units  
 (iii)  $\frac{x-3}{2} = \frac{4-y}{1} = \frac{z-2}{1}$   
 (iv)  $(\vec{r} - (3\hat{i} + 4\hat{j} + 2\hat{k})) \cdot (-2\hat{i} + \hat{j} - \hat{k}) = 0$   
 (v)  $\left(\frac{8}{3}, \frac{25}{6}, \frac{11}{6}\right)$

3. (i)  $51x + 15y - 50z + 173 = 0$   
 (ii)  $5x + 3y + 6z + 5 = 0$   
 (iii)  $(1, 0, 3)$   
 (iv) 0 units  
 (v)  $2x + y - z + \sqrt{6} = 0$

## **CHAPTER: 13 PROBABILITY**

### **SHORT ANSWER TYPE QUESTIONS-I (SA-I) (2 MARKS)**

Q.1 It is given that the event A and B are such that  $P(A) = \frac{1}{4}$ ,  $P(A/B) = \frac{1}{2}$  and  $P(B/A) = \frac{2}{3}$ , then find  $P(B)$

Q.2 If  $P(A \cap \bar{B}) = 0.15$ ,  $P(B) = 0.10$ , then find the value of  $P(A/\bar{B})$

Q.3 If A and B are two events with  $P(A) = \frac{3}{8}$ ,  $P(B) = \frac{5}{8}$ , and  $P(A \cup B) = \frac{3}{4}$  then find the value of  $P(A/B) \cdot P(A'/B)$ .

Q.4 If A and B are two independent events with  $P(A) = \frac{3}{5}$  and  $P(B) = \frac{4}{9}$ , then find the value of  $P(A' \cap B')$

Q.5 The following probability is distribution of random variable X. Find  $x$ .

X	0	1	2	3
$P(X)$	$\frac{1}{10}$	$x$	$\frac{3}{10}$	$\frac{4}{10}$

Q.6 The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, then calculate  $P(\bar{A}) + P(\bar{B})$

Q.7 A random variable X has the following distribution table:

X	1	2	3	4
$P(X)$	$k$	$2k$	$3k$	$4k$

Determine  $P(X < 3)$

Q.8 Two events E and F are independent. If  $P(E) = \frac{1}{2}$  and  $P(F) = \frac{1}{3}$ , find  $P(\text{neither } E \text{ nor } F)$

Q.9 Given that E and F are events such that  $P(E) = 0.8$ ,  $P(F) = 0.7$ ,  $P(E \cap F) = 0.6$ . Find  $P(\bar{E}/\bar{F})$

Q.10 If A and B are two independent events with  $P(A) = \frac{1}{3}$  and  $P(B) = \frac{1}{4}$ , then find  $P(B'/A)$

Q.11 If a leap year is selected at random, then what is the chance that it will contain 53 Tuesday?

Q.12 10 % of the bulbs produced in a factory are red colour and 2% are red and defective. If one bulb is picked at random, determine the probability of its being defective if it is red?

Q.13 A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event “number obtained is even” and B be the event “number obtained is red”. Find if A and B are independent events.

## SHORT ANSWER TYPE QUESTIONS-II (SA-II) (3 MARKS)

Q.1 Find the probability distribution of the maximum of the two scores obtained when a pair of die is thrown twice.

Q.2 Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one-by- one with replacement.

Q.3 A coin is tossed twice. Find the probability distribution of number of heads.

Q.4 If  $P(B) = \frac{3}{5}$ ,  $P(A/B) = \frac{1}{2}$  and  $P(A \cup B) = \frac{4}{5}$ , then calculate  $P(A \cup B)' + P(A' \cup B)$

Q.5 A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?

Q.6 If A and B are two independent events with  $P(A) = 0.3$  and  $P(B) = 0.4$ , then find the value of (i)  $P(A \cap B)$  (ii)  $P(A \cup B)$  Ans: (i) 0.12 (ii) 0.58

Q.7 Prove that if E and F are independent events, then the events E and  $F'$  are also independent.

Q.8 Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz completion. Find the probability that 2 boys and 2 girls are selected.

Q.9 Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?

Q.10 From a set of 100 cards numbered 1 to 100, one card is drawn at random. Find the probability that the number on the card is divisible by 6 or 8, but not by 24.

Q.11 Probability of solving specific problem independently by A and B are  $\frac{1}{2}$  and  $\frac{1}{3}$  respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem.

Q.12 In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is the probability that a student chosen randomly studies in class XII given that the chosen student is a girl?

Q.13 The probabilities of two students X and Y coming to the school in time are  $\frac{5}{7}$  and  $\frac{2}{3}$  respectively. Assuming that the events, X coming in time, and Y coming in time, are independent, find the probability of only one of them coming to the school in time.

Q.14 A random variable  $X$  has the following distribution table:

X	0	1	2	3	4	5	6	7
$P(X)$	0	$k$	$2k$	$2k$	$3k$	$k^2$	$2k^2$	$7k^2 + k$

Determine (i)  $k$  (ii)  $P(X < 3)$  (III)  $P(X > 6)$  (iv)  $P(0 < X < 3)$

### LONG ANSWER TYPE QUESTIONS (4 MARKS)

Q.1 A card is drawn from a well shuffled deck of 52 cards. The outcome is noted, the card is replaced and the deck reshuffled. Another card is drawn from the deck. What is the probability that the first card is an ace and the second card is a red queen.

Q.2 From a pack of 52 playing cards, a card is lost. From the remaining 51 cards, two cards are drawn at random (without replacement) and are found to be both diamonds. What is the probability that the lost card was a card of heart?

Q.3 A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Q.4 An urn contains 3 red and 5 black balls. A ball is drawn at random; its colour is noted and returned to the urn. Moreover, 2 additional balls of the colour noted down, are put in the urn and then two balls are drawn at random (without replacement) from the urn. Find the probability that both the balls drawn are of red colour.

Q.5 Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event 'the die shows a number greater than 3' given that 'there is at least one head'.

Q.6 A coin is biased such that a head is three times as likely to occur as a tail. When it is tossed twice, then find the probability distribution of number of heads.

Q.7 A and B throw a die alternatively till one of them gets a number greater than four and wins the game. If A starts the game, what is the probability of B winning?

Q.8 A, B and C throw a pair of dice in that order alternatively till one of them gets a total of '9' and wins the game. Find their respective probabilities of winning, if A starts first.

Q.9 A die is thrown three times. Events A and B are defined as below:  
A: 5 on the first and 6 on the second throw. B: 3 or 4 on the third throw.  
Find the probability of B, given that A has already occurred.

Q.10 Two bags A and B contain 4 white and 3 black balls and 2 white and 2 black balls respectively. From bag A, two balls are drawn at random and then transferred to bag B. A ball is then drawn from bag B and is found to be a black ball. What is the probability that the transferred balls were 1 white and 1 black?

Q.11 A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.

Q.12 A man is known to speak truth 3 out of 5 times. He throws a die and reports that it is 4. Find the probability that it is actually a 4.

Q.13 There are three coins. One is a two – headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows head. What is the probability that it was the two – headed coin?

Q.14 Two numbers are selected at random (without replacement) from the first six positive integers. Let  $X$  denotes the larger of the two numbers obtained. Find the probability distribution of the random variable  $X$

Q.15 Suppose that the reliability of HIV Test is specified as follows: Reliability of people having HIV, 90% of the tests detect the disease but 10% go undetected. Reliability of people free of HIV, 99% of the test judged HIV–ve but 1% are diagnosed as showing HIV+ve. Form a large population of which 0.1% have HIV, one person is selected at random, given the HIV test and the pathologist reports him/her as HIV+ve. What is the probability that the person has actually HIV?

Q.16 A problem in Mathematics is given to 4 students A, B, C, D. Their chances of solving the problem, respectively, are  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ , and  $\frac{2}{3}$ . What is the probability that (i) the problem will be solved? (ii) at most one of them will solve the problem?

Q.17 A speaks truth in 70% of the cases and B speaks truth in 80 % of the cases .In what percentage of the cases: (i) they contradict each other in stating the same fact?(ii)they agree each other in stating the same fact?

Q.18 In a class, 5% of the boys and 10% of the girls have an IQ of more than 150. In the class 60% of the students are boys and rest girls. If a student is selected at random and found to have an IQ of more than 150, find the probability that the selected student is a boy.

Q.19 Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is at random drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Q.20 An urn contains 5 red and 5 black balls. A ball is drawn at random; its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability the second ball is red?

Q.21 A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee.

Q.22 A Bag I contains 5 red and 4 white balls and a Bag II contains 3 red and 3 white balls. Two balls are transferred from the Bag I to the Bag II and then one ball is drawn from the Bag II. If the ball drawn from the Bag II is red, then find the probability that one red and one white ball are transferred from the Bag I to the Bag II.

Q.23 Two cards are drawn successively with replacement from a well- shuffled pack of 52 playing cards. Find the probability distribution of number of kings.

Q.24 Two cards are drawn simultaneously (without replacement) from a well – shuffled pack of 52 playing cards. Find the probability distribution of the number of aces.

Q.25 A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, Find the probability of one of them being red and another black.

Q.26 A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope, just two consecutive letter TA are visible. What is the probability that the letter came from TATANAGAR?

Q.27 If A and B are two independent events such that  $P(\overline{A} \cap B) = \frac{2}{15}$  and  $P(A \cap \overline{B}) = \frac{1}{6}$ , then find  $P(A)$  and  $P(B)$

Q.28 A and B throw a pair of dice alternatively. A win the game if he gets a total of '7' and B wins the game if he gets a total of '10'. If A starts the game, then find the probability that B wins.

### **CASE STUDY BASED QUESTIONS**

Q.1 In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.



Based on the above information answer the following:

- (i) Find the conditional probability that an error is committed in processing given that Sonia processed the form.
- (ii) Find the probability that Sonia processed the form and committed an error.
- (iii) Find the total probability of committing an error in processing the form.
- (iv) The manager of the company wants to do quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is NOT processed by Vinay.

(v) Let  $A$  be the event of committing an error in processing the form and let  $E_1, E_2$  and  $E_3$  be the events that Vinay, Sonia and Iqbal processed the form. Then find the value of  $\sum_{i=1}^3 P(E_i/A)$

Q.2 An insurance company insures three types of vehicles i.e., type A, B and C. If it insured 12000 vehicles of type A, 16000 vehicles of type B and 20,000 vehicles of type C. Survey report says that the chances of their accident are 0.01, 0.03 and 0.04 respectively.



(Based on the information given above, write the answer of following)

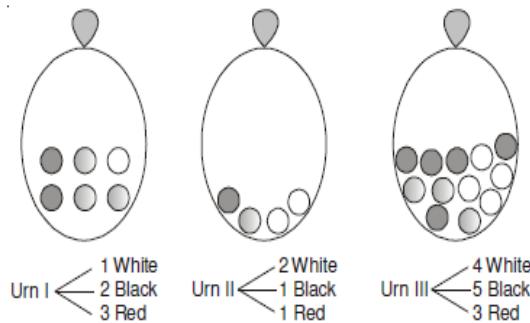
- (i) Find the probability of insured vehicle of type C.
- (ii) Let  $E$  be the event that insured vehicle meets with an accident, then find  $P(E/A)$ .
- (iii) Let  $E$  be the event that insured vehicle meets with an accident, then find  $P(E)$ .
- (iv) Find the probability of an accident that one of the insured vehicle meets with an accident and it is a type C vehicle.
- (v) Find the probability of one of the insured vehicles meets with an accident and it is not of type A and C.

Q.3 There are three Urns having different colored balls. The contents of Urns I, II, III are as follows:

Urn I: 1 white, 2 black and 3 red balls

Urn II: 2 white, 1 black and 1 red ball

Urn III: 4 white, 5 black and 3 red balls



Based on the above information answer the following questions:

- Find the probability that one white and one red ball is drawn only from Urn I.
- Find the probability of selecting any one of the urn.
- Using Baye's Theorem find the probability that balls are drawn from Urn I.
- Find the total probability of getting 1 white and 1 red ball.
- Find the probability that the balls are not drawn from III Urn.

Q.4 Ms. Manisha and Ms. Ritu are two friends. Ms. Manisha has 4 black and 6 red balls in her bag, where as Ms. Ritu has 7 black and 3 Red balls in her bag. They decided to throw a die and to draw the balls from their bags in such a way that, if 1 or 2 appears on die then ball will be drawn from Ms. Manisha's bag otherwise balls will be drawn from Ms. Ritu's bag.

On the basis of this situation answer the followings:



- Find the probability that Ms. Ritu's bag is not selected.
- Find the probability that Ms. Manisha's bag is selected.
- Find the probability if two balls are drawn at random (without replacement) in which 1 is red and 1 is black and drawn from Ms. Ritu's bag.

- (iv) Find the probability if two balls are drawn at random (without replacement) in which 1 is red and other black and drawn from Ms. Manisha's bag.
- (v) Find the total probability of drawing 1 red and 1 black ball.

Q.5 Three persons A, B and C apply for a job in a private school for the post of principal. The chances of their selection are in the ratio 2 : 3 : 4 respectively. Management committee given the agenda to improve the sports education, it is estimated that the change may occur with probability 0.8, 0.5 and 0.3 respectively.



On the bases of above situation answer the followings:

- (i) Find the probability that A is not selected.
- (ii) Find the probability of 'C' that change not take place.
- (iii) Find the probability of selection of C.
- (iv) Find the probability of 'A' that change occur.
- (v) Find the probability of 'B' that change not take place.

Q.6 By examine the test, the probability that a person is diagnosed with CORONA when he is actually suffering from it, is 0.99. The probability that the doctor incorrectly diagnosed a person to be having CORONA, on the basis of test reports, is 0.001. In a certain city, 1 in 1000 persons suffers from CORONA. A person is selected at random and is diagnosed to have CORONA. On the basis of above information, answer the following questions:

- (i) What is the **P(CORONA is diagnosed, when the person has not CORONA)** ?
- (ii) **P(CORONA is diagnosed, when the person actually has CORONA) ?**
- (iii) What is **P(CORONA is diagnosed) ?**
- (iv) What is the **P( Person has CORONA given CORONA is diagnosed) ?**

### ANSWERS KEY SA-I (2 MARKS)

1.  $\frac{1}{3}$   
 5.  $\frac{1}{5}$   
 9.  $\frac{1}{3}$   
 13. NO

2.  $\frac{1}{6}$   
 6. 1.1  
 10.  $\frac{3}{4}$

3.  $\frac{6}{25}$   
 7.  $\frac{3}{10}$   
 11.  $\frac{2}{7}$

4.  $\frac{2}{9}$   
 8.  $\frac{1}{3}$   
 12.  $\frac{1}{5}$

### ANSWERS KEY SA-II (3 MARKS)

1.

X	1	2	3	4	5	6
P(X)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

2.

X	0	1	2	3
P(X)	$\frac{343}{1000}$	$\frac{441}{1000}$	$\frac{189}{1000}$	$\frac{27}{1000}$

3.

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

6. (i) 0.12      6.(ii) 0.58

10.  $\frac{1}{5}$   
 13.  $\frac{9}{21}$

4. 1      5.  $\frac{1}{3}$   
 8.  $\frac{1}{14}$

11.  $\frac{2}{3}, \frac{1}{2}$   
 14.  $\frac{1}{10}, \frac{3}{10}, \frac{17}{100}, \frac{3}{10}$

### ANSWERS KEY LA ( 4 MARKS)

1.  $\frac{1}{338}$   
 4.  $\frac{1}{8}$

2.  $\frac{13}{50}$   
 5.  $\frac{1}{3}$

6.

X	0	1	2
P(X)	$\frac{1}{16}$	$\frac{6}{16}$	$\frac{9}{16}$

8.  $A = \frac{81}{217}, B = \frac{72}{217}, C = \frac{64}{217}$   
 11.  $\frac{63}{103}$

7.  $B = \frac{2}{5}, A = \frac{3}{5}$   
 9.  $\frac{1}{3}$

10.  $\frac{3}{5}$   
 13.  $\frac{20}{47}$

14.

X	2	3	4	5	6
P(X)	$\frac{2}{30}$	$\frac{4}{30}$	$\frac{6}{30}$	$\frac{8}{30}$	$\frac{10}{30}$

17. 38%, 62 %  
 20.  $\frac{1}{2}$

15. 0.0826  
 18.  $\frac{9}{17}$   
 21.  $\frac{126}{295}$

16.  $\frac{13}{15}, \frac{49}{90}$   
 19.  $\frac{16}{31}$   
 22.  $\frac{20}{37}$

$x$	0	1	2
$P(x)$	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

23.

$x$	0	1	2
$P(x)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

24.

25.  $\frac{22}{45}$

26.  $\frac{7}{11}$

27.  $P(A) = \frac{5}{6}$  and  $P(B) = \frac{4}{5}$  OR :  $P(A) = \frac{1}{5}$  and  $P(B) = \frac{1}{6}$

28.  $B = \frac{5}{17}$

### CASE STUDY BASED QUESTIONS ANSWERS KEY (4 MARKS)

1.	(i) 0.04	(ii) 0.008	(iii) 0.47	(iv) $\frac{17}{47}$	(v) 1
2.	(i) $\frac{5}{12}$	(ii) <b>0.01</b>	(iii) $\frac{34}{1200}$	(iv) $\frac{4}{7}$	(v) $\frac{12}{35}$
3.	(i) $\frac{1}{3}$	(ii) $\frac{1}{3}$	(iii) $\frac{33}{118}$	(iv) $\frac{118}{495}$	(v) $\frac{44}{59}$
4.	(i) $\frac{1}{3}$	(ii) $\frac{1}{3}$	(iii) $\frac{21}{45}$	(iv) $\frac{24}{45}$	(v) $\frac{22}{45}$
5.	(i) $\frac{7}{9}$	(ii) $\frac{7}{10}$	(iii) $\frac{4}{9}$	(iv) $\frac{8}{10}$	(v) $\frac{5}{10}$
6.	(i) 0.99	(ii) 0.001	(iii) 0.001989	(iv) $\frac{110}{221}$	

\*\*\*\*\*BEST WISHES\*\*\*\*\*

**KENDRIYA VIDYALAYA SANGATHAN, RAIPUR REGION**

**SAMPLE PAPER (SET-01)**

**TERM-2(2021-22)**

**CLASS-XII**

**SUBJECT-MATHEMATICS**

**BLUE PRINT**

<b>SL. NO</b>	<b>NAME OF THE CHAPTER</b>	<b>SA1(2)</b>	<b>SA2(3)</b>	<b>LA(4)</b>	<b>TOTAL</b>
1.	Integrals	1(2)	1(3)	1(4)	3(9)
2.	Applications of Integrals	-	-	1(4)	1(4)
3.	Differential Equations	1(2)	1(3)	-	2(5)
4.	Vectors	1(2)	1(3)	-	2(5)
5.	Three-Dimensional Geometry	1(2)	1(3)	1(4)	3(9)
6.	Probability	2(4)	-	1 CASE BASED (2+2)	3(8)
	<b>TOTAL</b>	<b>6(12)</b>	<b>4(12)</b>	<b>4(16)</b>	<b>14(40)</b>

# KENDRIYA VIDYALAYA SANGATHAN, RAIPUR REGION

## **SAMPLE PAPER (SET - 01)**

TERM-2 (2021-22)

CLASS-XII

## SUBJECT-MATHEMATICS

## TIME – 2 hours

## **MARKS-40**

## **GENERAL INSTRUCTIONS:**

1. This question paper contains 3 sections-A, B and C. Each part is compulsory.
2. Section-A has 6 short answer type (SA1) questions of 2 marks each.
3. Section-B has 4 short answer type (SA2) questions of 3 marks each.
4. Section-C has 4 long answer type (LA) questions of 4 marks each.
5. There is an internal choice in some of the questions.
6. Q 14 is a case-based problem having 2 subparts of 2 marks each.

S.No.	Question	Marks																		
<b><u>SECTION-A</u></b>																				
1.	<p>Evaluate: <math>\int_0^3 \frac{dx}{4+9x^2}</math></p> <p style="text-align: center;"><b>OR</b></p> <p>Evaluate: <math>\int \left( \frac{1+\sin x}{1+\cos x} \right) e^x dx</math></p>	2																		
2.	Write the order and the degree of the differential equation: $\frac{d^4y}{dx^4} + \sin\left(\frac{d^3y}{dx^3}\right) = 0$ .	2																		
3.	If $ \vec{a}  = 10$ , $ \vec{b}  = 2$ and $\vec{a} \cdot \vec{b} = 12$ , then find $ \vec{a} \times \vec{b} $ .	2																		
4.	If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular to each other, find the value of $k$	2																		
5.	<p>A random variable <math>X</math> has the following probability distribution:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr> <tr> <td>P(X)</td><td>0</td><td>k</td><td>2k</td><td>2k</td><td>3k</td><td><math>k^2</math></td><td><math>2k^2</math></td><td><math>7k^2 + k</math></td></tr> </table> <p>Determine (i) <math>k</math> (ii) <math>P(X &gt; 6)</math></p>	X	0	1	2	3	4	5	6	7	P(X)	0	k	2k	2k	3k	$k^2$	$2k^2$	$7k^2 + k$	2
X	0	1	2	3	4	5	6	7												
P(X)	0	k	2k	2k	3k	$k^2$	$2k^2$	$7k^2 + k$												
6.	A die is tossed thrice. Find the probability of getting an odd number at least	2																		

	once.	
<b><u>SECTION-B</u></b>		
7.	Evaluate: $\int \frac{1}{x(x^4-1)} dx$	3
8.	<p>Find the particular solution of the differential equation:</p> $(1 + x^2) \frac{dy}{dx} = e^{m \tan^{-1} x} - y, \text{ given that } y = 1 \text{ when } x = 0.$ <p style="text-align: center;"><b>OR</b></p> <p>Solve the differential equation: <math>\left[ x \sin^2 \left( \frac{y}{x} \right) - y \right] dx + x dy = 0, \text{ given } y = \frac{\pi}{4}</math> when <math>x = 1.</math></p>	3
9.	Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$ , $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ , and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$ . Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ , and $\vec{d} \cdot \vec{c} = 21.$	3
10.	<p>Find the shortest distance between the lines whose vector equations are</p> $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k}) \text{ and } \vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$ <p style="text-align: center;"><b>OR</b></p> <p>Find the vector equation of the plane which contains line of intersection of the planes <math>\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0</math> and <math>\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0</math> and which is perpendicular to the plane <math>\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0.</math></p>	3
<b><u>SECTION-C</u></b>		
11.	Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x \cosec x} dx$	4
12.	<p>Using integration, find the area of the smaller region bounded by the ellipse <math>4x^2 + 9y^2 = 36</math> and the line <math>2x + 3y = 6.</math></p> <p style="text-align: center;"><b>OR</b></p> <p>Using integration, find the area of the region bounded by the triangle whose vertices are <math>(1,0)</math>, <math>(2,2)</math> and <math>(3,1).</math></p>	4
13.	A plane meets the $x$ , $y$ and $z$ - axes at $A$ , $B$ and $C$ respectively, such that the centroid of the triangle $ABC$ is $(1, -2, 3).$ Find the Vector and Cartesian equations of the plane.	4
14.	<b><u>CASE BASED</u></b>	

	<p>Read the following text and answer the following questions on the basis of the same:</p> <p>The reliability of a COVID PCR test is specified as follows:</p> <p>Of people having COVID, 90% of the test detects the disease but 10% goes undetected. Of people free of COVID, 99% of the test judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/her COVID positive.</p> 	
(i)	What is the probability of the person is having actually COVID positive given that he is tested as COVID positive?	2
(ii)	What is the probability of the person selected will be diagnosed as COVID positive?	2

**KENDRIYA VIDYALAYA SANGATHAN, RAIPUR REGION**

**SAMPLE PAPER (SET - 01)**

**TERM-2 (2021-22)**

**CLASS-XII**

**SUBJECT-MATHEMATICS**

**MARKING SCHEME**

S.No.	Answers	Marks
1.	$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = \frac{1}{6} \left[ \tan^{-1} \frac{3x}{2} \right]_0^{\frac{2}{3}} \\ = \frac{\pi}{24}$ <p align="center"><b>OR</b></p> $\int \left( \frac{1+\sin x}{1+\cos x} \right) e^x dx = \int \left( \frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right) e^x dx \\ = e^x \tan \frac{x}{2} + c$	1 1 1 1 1
2.	<i>order = 4, degree is not defined.</i>	1+1
3.	$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{ \vec{a}   \vec{b} } = \frac{12}{20} = \frac{3}{5}$ , so $\sin \theta = \frac{4}{5}$ $ \vec{a} \times \vec{b}  =  \vec{a}   \vec{b}   \sin \theta  = 10 \times 2 \times \frac{4}{5} = 16.$	1 1
4.	$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0 \Rightarrow k = \frac{-10}{7}$	1+1
5.	$K = \frac{1}{10}$ $P(X > 6) = \frac{17}{100}$	1 1
6.	Probability of getting an even number three times = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ Probability of getting an odd number at least once = $1 - \frac{1}{8} = \frac{7}{8}$	1 1
7.	$\int \frac{1}{x(x^4 - 1)} dx = \int \frac{x^3}{x^4(x^4 - 1)} dx$ Let $x^4 = t \Rightarrow 4x^3 dx = dt$ $\int \frac{1}{x(x^4 - 1)} dx = \int \frac{x^3}{x^4(x^4 - 1)} dx = \frac{1}{4} \int \frac{1}{t(t-1)} dt$ $= \frac{1}{4} \int \left( \frac{1}{t-1} - \frac{1}{t} \right) dt$ $= \frac{1}{4} \log \left  \frac{x^4-1}{x^4} \right  + C$	1 1 1 1
8.	$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{m \tan^{-1} x}}{1+x^2}$ $P = \frac{1}{1+x^2}$ $Q = \frac{e^{m \tan^{-1} x}}{1+x^2}$ I.F = $e^{\tan^{-1} x}$ solution is	1 2 1 2 1 2

	$ye^{\tan^{-1}x} = \frac{e^{(m+1)\tan^{-1}x}}{m+1} + \frac{m}{m+1}$ <b>OR</b> $\frac{dy}{dx} = \frac{y}{x} - \sin^2 \frac{y}{x};$ Put $y = vx, \frac{dy}{dx} = v + x \frac{dv}{dx}$ To get $\int \frac{dv}{\sin^2 v} = - \int \frac{1}{x} dx$ $\Rightarrow -\cot \frac{y}{x} = -\log x  + c$ To get $c = -1$ To get the solution: $\cot \frac{y}{x} = \log x  + 1$ i.e., $\cot \frac{y}{x} = \log xe $	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
9.	A vector $\perp$ to both $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b} = 21\hat{i} - 21\hat{j} - 21\hat{k}$ Let $\vec{d} = \lambda(21\hat{i} - 21\hat{j} - 21\hat{k})$ ; $\vec{d} \cdot \vec{a} = 21 \Rightarrow 63\lambda - 21\lambda + 21\lambda = 21 \Rightarrow \lambda = \frac{1}{3}$ So, $\vec{d} = \frac{1}{3}(21\hat{i} - 21\hat{j} - 21\hat{k}) = 7(\hat{i} - \hat{j} - \hat{k})$ ;	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
10.	$\vec{a}_1 = \hat{i} + \hat{j}, \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}, \vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$ $\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$ $\vec{b}_1 \times \vec{b}_2 = 3\hat{i} - \hat{j} - 7\hat{k}$ and $ \vec{b}_1 \times \vec{b}_2  = \sqrt{59}$ $d = \frac{ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) }{ \vec{b}_1 \times \vec{b}_2 }$ $= \frac{10}{\sqrt{59}}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	<b>OR</b> Let the equation of the required plane be $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 + \lambda [\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5] = 0$ This is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$ . Therefore, $5(2\lambda + 1) + 3(\lambda + 2) - 6(3 - \lambda) = 0 \Rightarrow \lambda = \frac{7}{19}$ Therefore, vector equation of the plane is $\vec{r} \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) - 41 = 0$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
11.	$\int_0^\pi \frac{x \tan x}{\sec x \cosec x} dx = \int_0^\pi x \sin^2 x dx$ Let $I = \int_0^\pi x \sin^2 x dx = \int_0^\pi (\pi - x) \sin^2(\pi - x) dx$ $= \int_0^\pi (\pi - x) \sin^2 x dx$ $2I = \pi \int_0^\pi \sin^2 x dx = \pi \int_0^\pi \frac{1 - \cos 2x}{2} dx = \frac{\pi}{2} \left[ x - \frac{\sin 2x}{2} \right]_0^\pi = \frac{\pi^2}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

	$\therefore I = \frac{\pi^2}{4}$	$\frac{1}{2}$
12.	<p>To draw the correct graph</p> <p>Required area= <math>\int_0^3 \frac{\sqrt{36-4x^2}}{3} dx - \int_0^3 \frac{6-2x}{3} dx</math></p> $= \frac{3\pi}{2} - 3$ <p><b>OR</b></p> <p>To draw the graph and finding the equations of the sides</p> $y = 2(x - 1), \quad y = 4 - x, \quad y = \frac{1}{2}(x - 1)$ <p>Required area= <math>\int_1^2 2(x - 1) dx + \int_2^3 (4 - x) dx - \int_1^3 \frac{1}{2}(x - 1) dx</math></p> $= \frac{3}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
13.	<p>Let the coordinates of A, B and C be <math>(a, 0, 0), (0, b, 0)</math> and <math>(0, 0, c)</math> respectively.</p> <p>Therefore, the equation of plane is <math>\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1</math></p> <p>Here, <math>\frac{a+0+0}{3} = 1, \quad \frac{0+b+0}{3} = -2, \quad \frac{0+0+c}{3} = 3,</math></p> <p><math>\Rightarrow a = 3, \quad b = -6, \quad c = 9</math></p> <p>Therefore, the equation of plane is <math>\frac{x}{3} + \frac{y}{-6} + \frac{z}{9} = 1</math>  <i>i.e.,</i> <math>6x - 3y + 2z - 18 = 0</math></p> <p>which in vector form is <math>\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 18</math></p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
14.	<p><math>E_1</math>: Person actually having COVID, <math>E_2</math>: Person actually not having COVID and A: person tested as positive</p> <p>(i) <math>P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)} = 0.0826</math></p> <p>(ii) <math>P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) = 0.01089</math></p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

**KENDRIYA VIDYALAYA SANGATHAN, REGIONAL OFFICE RAIPUR**

**SAMPLE PAPER (SET – 02)**

**TERM –2 (2021-22)**

**CLASS –XII**

**SUB: MATHEMATICS**

**BLUE-PRINT**

<b>S.No.</b>	<b>Chapter</b>	<b>SA-1 (2 Marks)</b>	<b>SA-2 (3 Marks)</b>	<b>LA (4 Marks)</b>	<b>Total</b>
1	INTEGRAL	1(2)	1(3)	1(4)	<b>3(9)</b>
2	APPLICATIONS OF INTEGRALS			1(4)	<b>1(4)</b>
3	DIFFERENTIAL EQUATION	1(2)	1(3)	-	<b>2(5)</b>
4	VECTOR ALGEBRA	1(2)	1(3)	-	<b>2(5)</b>
5	3-D GEOMETRY	1(2)	1(3)	1(4)	<b>3(9)</b>
6	PROBABILITY	2(4)	-	1(4)	<b>3(8)</b>
<b>TOTAL</b>		<b>6(12)</b>	<b>4(12)</b>	<b>4(16)</b>	<b>14(40)</b>

**KENDRIYA VIDYALAYA SANGATHAN, REGIONAL OFFICE RAIPUR**

**SAMPLE PAPER (SET- 02)**

**TERM -2 (2021-22)**

**CLASS – XII**

**SUB: MATHEMATICS**

**(041)**

**TIME- 2 hours**

**M.M:40**

**General Instructions:**

1. This question paper contains **three sections–A, B and C**. Each part is compulsory.
2. **Section – A** has **6 short answer type (SA1) questions** of **2 marks each**.
3. **Section–B** has **4 short answer type (SA2) questions** of **3marks each**.
4. **Section – C** has **4 long answer type questions (LA)** of **4marks each**.
5. There is an **internal choice** in some of the questions.
6. Q14 is a **case-based problem** having 2 subparts of 2 marks each.

<b>S.No.</b>	<b>Question</b>	<b>Marks</b>
	<b><u>SECTION–A</u></b>	
1.	Find $\int \frac{\tan^3 x}{\cos^3 x} dx$ <b>OR</b> Find $\int \sin x \log \cos x dx$	2
2	Find the sum of degree and order of differential equation $x^2 \frac{d^2 y}{dx^2} = 1 + \left[ \left( \frac{dy}{dx} \right)^2 \right]^4$	2
3	Find the unit vector perpendicular to each of the vectors : $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$	2
4	Show that the plane $x - 5y - 2z = 1$ contains the line $\frac{x-5}{3} = y = 2 - z$	2
5	A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2balls are drawn at random from the bag one-by-one without replacement.	2
6	Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card Jack.	2
	<b><u>SECTION B</u></b>	
7	Find $\int \sec^3 x dx$	3

8	<p>Find the general solution of the following differential equation:  <math>xdy - (y + 2x^2)dx = 0</math></p> <p><b>OR</b></p> <p>Find the particular solution of the following differential equation:  <math>\frac{dy}{dx} = 1+x^2+y^2 + x^2y^2</math> given that <math>y = 1</math> when <math>x = 0</math></p>	3
9	If $\vec{a} \neq \vec{0}$ , $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ , $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ , then show that $\vec{b} = \vec{c}$ .	3
10	<p>Find the shortest distance between the following lines:</p> $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + s(2\hat{i} + \hat{j} + \hat{k})$ $\vec{r} = (\hat{i} + \hat{j} + 2\hat{k}) + t(4\hat{i} + 2\hat{j} + 2\hat{k})$	3
	<b>OR</b>	
	<p>Find the vector and the Cartesian equations of the plane containing the point <math>\hat{i} + 2\hat{j} - \hat{k}</math> and parallel to the lines</p> $\vec{r} = (\hat{i} + 2\hat{j} + 2\hat{k}) + s(2\hat{i} - 3\hat{j} + 2\hat{k}) = 0$ and $\vec{r} = (3\hat{i} + \hat{j} - 2\hat{k}) + t(\hat{i} - 3\hat{j} + \hat{k}) = 0$	
	<b>SECTION C</b>	
11	Evaluate : $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$	4
12	<p>Using integration, find the area of the region in the first quadrant enclosed by the line <math>x + y = 2</math>, the parabola <math>y^2 = x</math> and the x-axis.</p> <p><b>OR</b></p> <p>Using integration, find the area of the region</p> $\{(x,y): 0 \leq y \leq 3\sqrt{x}, x^2 + y^2 \leq 4\}$	4
13	Find the foot of the perpendicular from the point $(1,2,0)$ upon the plane $x - 3y + 2z = 9$ . Hence, find the distance of the point $(1,2,0)$ from the given plane.	4

14

**CASE STUDY**

A shopkeeper sells three types of flower seeds  $A_1$ ,  $A_2$  and  $A_3$ . They are sold as mixture where proportions are 4:4:2 respectively. Their germination rates are 45%, 60% and 35% respectively. Calculate the probability

(i) of a randomly chosen seed to germinate  
(ii) that it is of the type  $A_2$  given that a randomly chosen seed does not germinate.

2  
2

**KENDRIYA VIDYALAYA SANGATHAN, REGIONAL OFFICE RAIPUR**

**SAMPLE PAPER (SET – 02)**

**TERM -2(2021-22)**

**CLASS – XII**

**SUB-MATHEMATICS**

**MARKING SCHEME**

S.No.	ANSWE RS	MARKS						
1	$I = \int \frac{\tan^3 x}{\cos^3 x} dx = \int \tan^3 x \sec^3 x dx, \text{put } \tan x = t, I = \int t^3 \sqrt{1+t^2} dt, \text{put}$ $1+t^2 = u, I = \int \sqrt{u} (u-1) \frac{du}{2}$ $I = \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + c$ <p style="text-align: center;">OR</p> $\text{Put } \cos x = t, I = - \int \log t dt$ $= t(t-1) + c = \cos x (1 - \log(\cos x)) + c$	1 1 1 1 1						
2	Order = 2, degree = 1, sum = 2	2						
3	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 1 \\ 2 & -1 & 2 \end{vmatrix} = 7\hat{i} - 6\hat{j} - 10\hat{k},$ $ \vec{a} \times \vec{b}  = \sqrt{185}, \hat{n} = \frac{1}{\sqrt{185}}$	1 1						
4	d.r.s. of line 3, 1, -1, d.r.s of normal to the plane 1, -5, -2 $3(1) + 1(-5) + (-1)(-2) = 0$	1 1						
5	<p>:Let X be the random variable defined as the number of red balls.  Then <math>X = 0, 1</math></p> $P(X=0) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$ $P(X=1) = \frac{1}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{1}{3} = \frac{1}{2}$ <p>Probability Distribution Table:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>X</td><td>1</td><td>2</td></tr> <tr> <td>P (X)</td><td><math>\frac{1}{2}</math></td><td><math>\frac{1}{2}</math></td></tr> </table>	X	1	2	P (X)	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
X	1	2						
P (X)	$\frac{1}{2}$	$\frac{1}{2}$						
6	The required probability = $P((\text{The first is a red jack card and The second is a jack card}) \text{ or } (\text{The first is a red non-jack card and The second is a jack card}))$ $= \frac{2}{52} \times \frac{3}{51} + \frac{24}{52} \times \frac{4}{51} = \frac{1}{26}$	1 1						
7	$Let I = \int \sec^3 x dx = \int \sec x \sec^2 x dx = \sec x \tan x - \int \sec x \tan^2 x dx$ $= \sec x \tan x - \int \sec^3 x dx + \int \sec x dx$	1 1						

	$2I = \sec x \tan x + \log \sec x + \tan x  + c'$ $I = \frac{\sec x \tan x}{2} + \frac{1}{2} \log \sec x + \tan x  + c$	1
8	$\frac{dy}{dx} = \frac{y+2x^2}{x}$ $\frac{dy}{dx} - \frac{y}{x} = 2x$ $I.F = e^{-\int \frac{1}{x} dx}$ $y = 2x^2 + cx$ <p style="text-align: center;">OR</p> $\frac{dy}{dx} = (1+x^2)(1+y^2)$ $\frac{dy}{(1+y^2)} = (1+x^2)dx$ $\int \frac{dy}{(1+y^2)} dx = \int (1+x^2) dx + c$ $\tan^{-1} y = \frac{x^3}{3} + x + c$ $c = \tan^{-1} 1 = \frac{\pi}{4}$ $\tan^{-1} y = \frac{x^3}{3} + x + \frac{\pi}{4}$	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
9	<p>Solution: We have <math>\vec{a} \cdot (\vec{b} - \vec{c}) = 0</math></p> $\Rightarrow (\vec{b} - \vec{c}) = \vec{0} \text{ or } \vec{a} \perp (\vec{b} - \vec{c})$ $\Rightarrow \vec{b} = \vec{c} \text{ or } \vec{a} \perp (\vec{b} - \vec{c})$ <p>Also, <math>\vec{a} \times (\vec{b} - \vec{c}) = \vec{0}</math></p> $\Rightarrow (\vec{b} - \vec{c}) = \vec{0} \text{ or } \vec{a} \parallel (\vec{b} - \vec{c})$ $\Rightarrow \vec{b} = \vec{c} \text{ or } \vec{a} \parallel (\vec{b} - \vec{c})$ $\Rightarrow \vec{b} = \vec{c} \text{ or } \vec{a} \parallel (\vec{b} - \vec{c})$ <p><math>\vec{a}</math> cannot be both perpendicular to <math>(\vec{b} - \vec{c})</math> and parallel to <math>(\vec{b} - \vec{c})</math></p> <p>Hence, <math>\vec{a} = \vec{c}</math>.</p>	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
10	<p>The lines are parallel. The shortest distance = <math>\frac{ (\vec{a} - \vec{b}) \times \vec{a} }{ \vec{a} }</math></p> $= \frac{ (3\hat{k}) \times (2\hat{i} + \hat{j} + \hat{k}) }{\sqrt{4+1+1}}$ $(3\hat{k}) \times (2\hat{i} + \hat{j} + \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix} = -3\hat{i} + 6\hat{j}$ <p>Hence, the required shortest distance = <math>\frac{3\sqrt{5}}{\sqrt{6}}</math> units</p>	$1\frac{1}{2}$ $1\frac{1}{2}$ $1/2$ $1/2$ $1/2$

OR

Since, the plane is parallel to the given lines, the cross product of the vectors  $2\hat{i} - 3\hat{j} + 2\hat{k}$  and  $\hat{i} - 3\hat{j} + \hat{k}$  will be a normal to the plane

$$(2\hat{i} - 3\hat{j} + 2\hat{k}) \times (\hat{i} - 3\hat{j} + \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ 1 & -3 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{k}$$

The vector equation of the plane is  $\vec{r} \cdot (3\hat{i} - 3\hat{k}) = (\hat{i} + 2\hat{j} - \hat{k}) \cdot (3\hat{i} - 3\hat{k})$

$$\text{or, } \vec{r} \cdot (\hat{i} - \hat{k}) = 2$$

and the Cartesian equation of the plane is  $x - z - 2 = 0$

11

$$\begin{aligned} I &= \int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx \\ &= \int_0^\pi \frac{x \sin x}{1 + \sin x} dx \\ &= \int_0^\pi \frac{(\pi - x) \sin x}{1 + \sin x} dx \\ 2I &= \pi \int_0^\pi \frac{\sin x}{1 + \sin x} dx \\ &= \pi \int_0^\pi (\tan x \sec x - \tan^2 x) dx \\ &= \pi [\sec x - \tan x + x]_0^\pi \\ &= \frac{\pi}{2} (\pi - 2) \end{aligned}$$

$\frac{1}{2}$

$\frac{1}{2}$

1

12

Solving  $x + y = 2$  and  $y^2 = x$  to get point of intersection as  $(1, 1)$  and  $(4, 2)$

Correct fig

Reqd area  $\int_0^1 \sqrt{x} dx + \int_1^2 (2 - x) dx$

$$\begin{aligned} &= \frac{2}{3} \left[ x^{\frac{3}{2}} \right]_0^1 + \left[ 2x - \frac{x^2}{2} \right]_1^2 \\ &= \frac{2}{3} + \frac{1}{2} = \frac{7}{6} \text{ sq units} \end{aligned}$$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

OR

Solving  $y = \sqrt{3}x$  and  $x^2 + y^2 = 4$  to get point of intersection as  $(1, \sqrt{3})$  and  $(-1, -\sqrt{3})$

Correct fig

Reqd area  $= \int_0^1 \sqrt{3}x dx + \int_1^2 \sqrt{4 - x^2} dx$

$$\begin{aligned} &= \frac{\sqrt{3}}{2} [x^2]_0^1 + \frac{1}{2} \left[ x \sqrt{4 - x^2} + 4 \sin^{-1} \frac{x}{2} \right]_1^2 \\ &= \frac{2\pi}{3} \text{ sq units} \end{aligned}$$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

13	<p>: The equation of the line perpendicular to the plane and passing through the point (1, 2, 0) is</p> $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z}{2}$ <p>The coordinates of the foot of the perpendicular are <math>(\mu + 1, -3\mu + 2, 2\mu)</math> for some <math>\mu</math></p> <p>These coordinates will satisfy the equation of the plane. Hence, we have</p> $\mu + 1 - 3(-3\mu + 2) + 2(2\mu) = 9$ $\Rightarrow \mu = 1$ <p>The foot of the perpendicular is (2, -1, 2).</p> <p>Hence, the required distance = <math>\sqrt{14}</math> units</p>	1 $\frac{1}{2}$ 1 $\frac{1}{2}$ 1
14	<p>Let <math>A_1</math>: seed <math>A_1</math> is chosen, <math>A_2</math>: seed <math>A_2</math> is chosen &amp; <math>A_3</math>: seed <math>A_3</math> is chosen</p> <p><math>E</math>: seed germinates and <math>\bar{E}</math>: seed germinates</p> $P(A_1) = \frac{4}{10}, P(A_2) = \frac{4}{10}, P(A_3) = \frac{2}{10}, P(E/A_1) = \frac{45}{100}, P(E/A_2) = \frac{60}{100},$ $P(E/A_3) = \frac{35}{100}$ <p>(i) <math>P(E) = P(A_1)P\left(\frac{E}{A_1}\right) + P(A_2)P\left(\frac{E}{A_2}\right) + P(A_3)P\left(\frac{E}{A_3}\right) = 0.49</math></p> <p>(ii) <math>P(A_2/\bar{E}) = \frac{P(A_2)P\left(\frac{\bar{E}}{A_2}\right)}{P(A_1)P\left(\frac{\bar{E}}{A_1}\right) + P(A_2)P\left(\frac{\bar{E}}{A_2}\right) + P(A_3)P\left(\frac{\bar{E}}{A_3}\right)} = \frac{16}{51}</math></p>	3 2 2

**KENDRIYA VIDYALAYA SANGATHAN, RAIPUR REGION**

**SAMPLE PAPER (SET-03)**

**TERM-2(2021-22)**

**CLASS-XII**

**SUBJECT-MATHEMATICS**

**BLUE - PRINT**

<b>SL. NO</b>	<b>NAME OF THE CHAPTER</b>	<b>SA1(2)</b>	<b>SA2(3)</b>	<b>LA(4)</b>	<b>TOTAL</b>
1.	Integrals	1(2)	1(3)	1(4)	3(9)
2.	Applications of Integrals	-	-	1(4)	1(4)
3.	Differential Equations	1(2)	1(3)	-	2(5)
4.	Vectors	1(2)	1(3)	-	2(5)
5.	Three-Dimensional Geometry	1(2)	1(3)	1(4)	3(9)
6.	Probability	2(4)	-	1 CASE BASED (2+2)	3(8)
	<b>TOTAL</b>	<b>6(12)</b>	<b>4(12)</b>	<b>4(16)</b>	<b>14(40)</b>

**KENDRIYA VIDYALAYA SANGATHAN, RAIPUR REGION**

**SAMPLE PAPER (SET - 03)**

**TERM-2 (2021-22)**

**CLASS-XII**

**SUBJECT-MATHEMATICS**

**TIME – 2 HOURS**

**MARKS-40**

**GENERAL INSTRUCTIONS:**

1. This question paper contains 3 sections-A, B and C. Each part is compulsory.
2. Section-A has 6 short answer type (SA1) questions of 2 marks each.
3. Section-B has 4 short answer type (SA2) questions of 3 marks each.
4. Section-C has 4 long answer type (LA) questions of 4 marks each.
5. There is an internal choice in some of the questions.
6. Q 14 is a case-based problem having 2 subparts of 2 marks each.

S.No.	Question	Marks
<b><u>SECTION-A</u></b>		
1.	Evaluate: $\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}} dx, \quad 0 < x < \frac{\pi}{2}$ <b>OR</b> Evaluate $\int \frac{dx}{\sin^2 x \cos^2 x}$	2
2.	Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$ .	2
3.	Solve: $\frac{dy}{dx} + 2y = e^{3x}$	2
4.	Find the cartesian equation of the line which passes through the point (-2,4,-5) and parallel to the line $\frac{x-1}{-2} = \frac{1-y}{3} = \frac{3-z}{-4}$	2
5.	A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement, find the probability that none is red .	2
6.	If A and B are two events such that $P(A) = \frac{1}{4}$ , $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$ . Find $P(\text{not } A \text{ and not } B)$ .	2
<b><u>SECTION-B</u></b>		
7.	Evaluate : $\int \frac{2x}{(1+x^2)(3+x^2)} dx$	3

8.	<p>Solve the differential equation: <math>x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x</math></p> <p><b>OR</b></p> <p>Solve the differential equation:</p> $(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$	3
9.	If $\hat{a}$ and $\hat{b}$ are two unit vectors and $\theta$ is the angle between them prove that $\sin\frac{\theta}{2} = \frac{1}{2}  \hat{a} - \hat{b} $ .	3
10.	<p>Find the shortest distance between the two skew lines</p> $\vec{r} = 8\hat{i} - 9\hat{j} + 10\hat{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$ <p><b>OR</b></p> <p>Find the length and foot of the perpendicular from the point <math>(1, 3/2, 2)</math> to the plane <math>2x - 2y + 4z = 0</math>.</p>	3
<b>SECTION-C</b>		
11.	Evaluate : $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$	4
12.	<p>Find the area bounded by the curve <math>y = x^2</math> and the lines <math>y=4</math>.</p> <p><b>OR</b></p> <p>Find the area of the region included between the parabola <math>y = \frac{3x^2}{4}</math> and the line <math>3x - 2y + 12 = 0</math>.</p>	4
13.	Find the vector equation of the line passing through $(1, 2, 3)$ and parallel to the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + 3\hat{j} + \hat{k}) = 6$ .	4
14.	<p>Let <math>X</math> denote the number of college where you will apply after yours result and <math>P(X=x)</math> denotes your probability of getting admission in <math>x</math> number of college. It is given that</p> $P(X=x) = \begin{cases} kx, & \text{if } x = 0 \text{ or } 1 \\ 2kx, & \text{if } x = 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{if } x > 4 \end{cases}$ <p>Where <math>k</math> is a positive constant.</p> <p>Based on the above information answer the following:</p>	
i).	Find the value of $k$ .	2
ii).	What is the probability that you will get admission in at least two college.	2

**KENDRIYA VIDYALAYA SANGATHAN, RAIPUR REGION**

**SAMPLE PAPER (SET - 03)**

**TERM-2 (2021-22)**

**CLASS-XII**

**SUBJECT-MATHEMATICS**

**MARKING SCHEME**

<b>S.No.</b>	<b>Answers</b>	<b>Marks</b>
1.	$\sqrt{1 + \sin 2x} = \sqrt{(\sin x + \cos x)^2} = \sin x + \cos x$ $\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}} dx = \int \frac{-dt}{t}$ $= -\log t + C = -\log(\sin x + \cos x) + C$ <p>OR</p> <p>For writing <math>\int \frac{(\sin^2 x + \cos^2 x) dx}{\sin^2 x \cos^2 x}</math> and separating it</p> $I = \tan x - \cot x + C$	1 $\frac{1}{2}$ $\frac{1}{2}$ 1 1
2.	Direction ratios of the vector 1,2,3 For finding direction cosines $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$	$\frac{1}{2}$ $1\frac{1}{2}$
3.	I. F. = $e^{2x}$ For finding correct solution $ye^{2x} = \frac{e^{5x}}{5} + C$	$\frac{1}{2}$ $1\frac{1}{2}$
4.	Direction ratios of the line -2,-3,4 Cartesian equation of the line $\frac{x+2}{-2} = \frac{y-4}{-3} = \frac{z+5}{4}$	$\frac{1}{2}$ $1\frac{1}{2}$
5.	$P(\text{none is red}) = \frac{8}{15} \times \frac{7}{14} \times \frac{6}{13}$ $= \frac{8}{65}$	$1\frac{1}{2}$ $\frac{1}{2}$
6.	$P(A) = \frac{1}{4}$ , $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$ $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$ $P(\text{not } A \text{ and not } B) = P(A' \cap B') = 1 - P(A \cup B) = \frac{3}{8}$	1 1
7.	$I = \int \frac{2x}{(1+x^2)(3+x^2)} dx$ Let $x^2 = t$ $dt = 2x dx$ $I = \int \frac{dt}{(1+t)(3+t)}$ Getting result by partial function	$\frac{1}{2}$ $\frac{1}{2}$ 2
8.	For taking $y = vx$ and finding $\frac{dy}{dx} = v + x \frac{dv}{dx}$ For finding $x \frac{dv}{dx} = 1/\cos v$ For finding correct solution $\sin(\frac{y}{x}) = \log cx $ OR $(x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$ $\frac{dy}{dx} + \frac{2xy}{x^2 + 1} = \frac{\sqrt{x^2 + 4}}{x^2 + 1}$ is a linear differential equation	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$ 2

	Getting integrating factor = $x^2+1$ Multiplying I.F. and finding solution	
9.	$ \hat{a} - \hat{b} ^2 =  \hat{a} ^2 +  \hat{b} ^2 - 2 \hat{a} \cdot \hat{b}$ $= 1 + 1 - 2 \cos \theta$ $= 4 \sin^2 \frac{\theta}{2}$ Showing result	1 1 $\frac{1}{2}$ $\frac{1}{2}$
10.	Shortest distance = $\frac{(\vec{a2} - \vec{a1}) \cdot (\vec{b1} \times \vec{b2})}{ \vec{b1} \times \vec{b2} }$ Finding $\vec{a2} - \vec{a1}$ and $\vec{b1} \times \vec{b2}$ $ \vec{b1} \times \vec{b2} $ Getting result OR $\frac{x-1}{2} = \frac{y-3/2}{-2} = \frac{z-2}{4} = k$ $X = 2k+1 \quad y = -2k+3/2 \quad z = 4k+2$ Putting these values in equation of plane and finding the value of K For finding Foot = (x, y, z) For finding length	1 1 1 $\frac{1}{2}$ 1 1 $\frac{1}{2}$
11.	Let $I = \int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx = \int_0^\pi \frac{(\pi-x) \tan(\pi-x)}{\sec(\pi-x) + \tan(\pi-x)} dx = \int_0^\pi \frac{(\pi-x) \tan x}{\sec x + \tan x} dx$ $2I = \pi \int_0^\pi \frac{\tan x}{\sec x + \tan x} dx$ Getting result $I = \frac{\pi(\pi-2)}{2}$	1 1 2
12.	For correct figure Required area = $2 \int_0^4 x dy$ $= 2 \int_0^4 \sqrt{y} dy$ $= 32/3$ sq unit OR For finding limit x=-2 to x=4 For correct figure $A = \int_{-2}^4 \frac{12+3x}{2} dx - \int_{-2}^4 \frac{3x^2}{4} dx$ For integrating and writing correct area 27 sq unit.	1 1 $\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 2
13.	Let a, b and c be the drs of the req. line $a - b + 2c = 0$ and $3a + 3b + c = 0$ finding a, b and c in the terms of arbitrary constant $a = -7k$ , $b = 5k$ , $c = 6k$ finding vector equation of line $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-7\hat{i} + 5\hat{j} + 6\hat{k})$	2 1 1
14.	i) $k+4k+2k+k = 1$ $K=1/8$ ii) $P(\text{getting admission in atleast two colleges}) =$ $4k+2k+k$ $= 7k = 7/8$	1 1 1 1